Curs 4

2026/2027

Dispozitive şi circuite de microunde pentru radiocomunicaţii

Disciplina 2026/2027

- 2C/1L/1P, **DCMR (CDM)**
- **Minim 14 prezente (curs+laborator+proiect)**
- Curs - **conf. Radu Damian**
  - Vineri 8-10/**Video (istoric)**, C1, corp C
  - E – **40%** din nota
  - **probleme** + (2p prez. curs) + (3 teste) + (bonus activitate)
    - primul test L1: ~01.10.2025 (t2 si t3 neanuntate la **curs**)
    - 3pz (C) ≈ +0.5p (**2p** max)
  - **toate materialele permise**

Disciplina 2026/2027

- 2C/1L/1P, **DCMR (CDM)**
- Laborator – **conf. Radu Damian**
  - Miercuri 08-16, **II.13**
  - Joi 12-20, **I.08**
  - L – **30%** din nota
    - ADS, tema personala
    - prezenta + **rezultate personale!**
  - P – **30%** din nota
    - ADS, tema personala
  - S1÷14: 3P, 7L, 4P
    - prezenta cu alte grupe **permisa** in limita numarului de calculatoare disponibile

Documentatie

- https://rf-opto.etti.tuiasi.ro



Scop curs 4





Sinapse "inginerești"

Cuprins

- Linii de transmisie
- Adaptarea de impedanța
- Cuploare direcționale
- Divizoare de putere
- Amplificatoare de microunde
- Filtre de microunde
- Oscilatoare de microunde ?

Examen: Reprezentare logaritmică

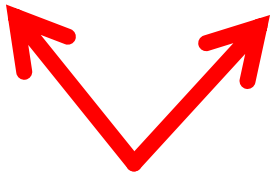
- $\boldsymbol{X[dB] = 10 \cdot log_{10}(X[lin])}$
- $X[B] = log_{10}(X[lin])$
- X[lin] – adimensional
- $log(A \cdot B) = log(A) + log(B)$

- $\boldsymbol{X[lin] = 10^{X[dB]/10}}$

Calculul atenuarii/amplificarii

Pierderi/Atenuare $\rightarrow P_{out} < P_{in} \rightarrow P_{out}[\text{dBm}] < P_{in}[\text{dBm}]$

$$P_{out}[\text{dBm}] = P_{in}[\text{dBm}] - \text{Pierderi/Atenuare}[\text{dB}]$$



Castig/Amplificare $\rightarrow P_{out} > P_{in} \rightarrow P_{out}[\text{dBm}] > P_{in}[\text{dBm}]$

$$P_{out}[\text{dBm}] = P_{in}[\text{dBm}] + \text{Castig/Amplificare}[\text{dB}]$$



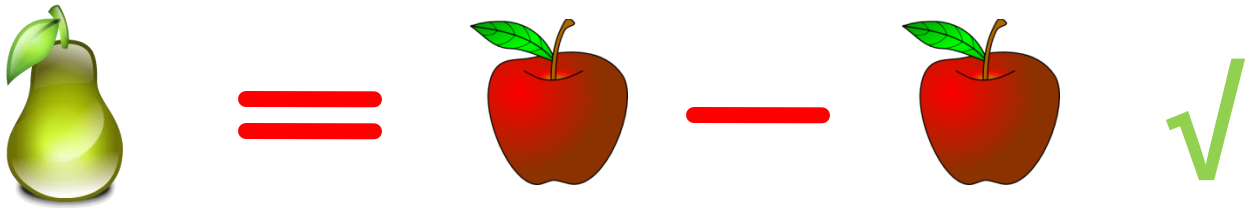
Calculul atenuarii/amplificarii

$$\text{Pierderi/Castig} = \frac{P_{out}}{P_{in}}$$

$$\text{Pierderi}[\text{dB}] = [-]10 \cdot \log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

$$\text{Pierderi}[\text{dB}] = [-]10 \cdot \log_{10}\left(\frac{P_{out}}{P_0} \cdot \frac{P_0}{P_{in}}\right) = [-]10 \cdot \left[\log_{10}\left(\frac{P_{out}}{P_0}\right) - \log_{10}\left(\frac{P_{in}}{P_0}\right)\right]$$

$$\text{Pierderi}[\text{dB}] = [-]\left(P_{out}[\text{dBm}] - P_{in}[\text{dBm}]\right)$$



Examen

- Operatii cu numere complexe!
- $z = a + j \cdot b$; $\mathbf{j^2 = -1}$

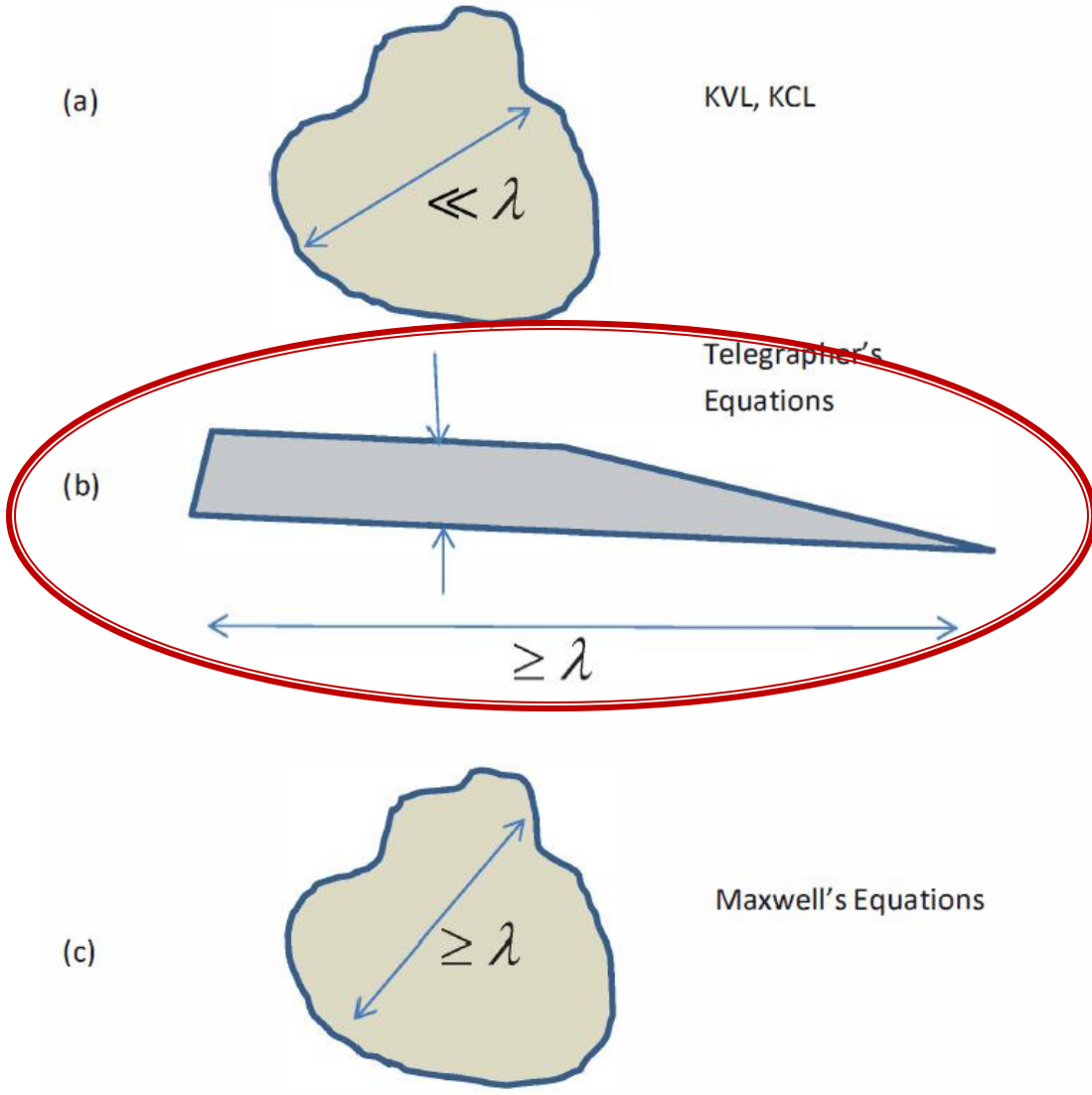
Cuprins

- Linii de transmisie
- Adaptarea de impedanța
- Cuploare direcționale
- Divizoare de putere
- Amplificatoare de microunde
- Filtre de microunde
- Oscilatoare de microunde ?

Lungime electrica

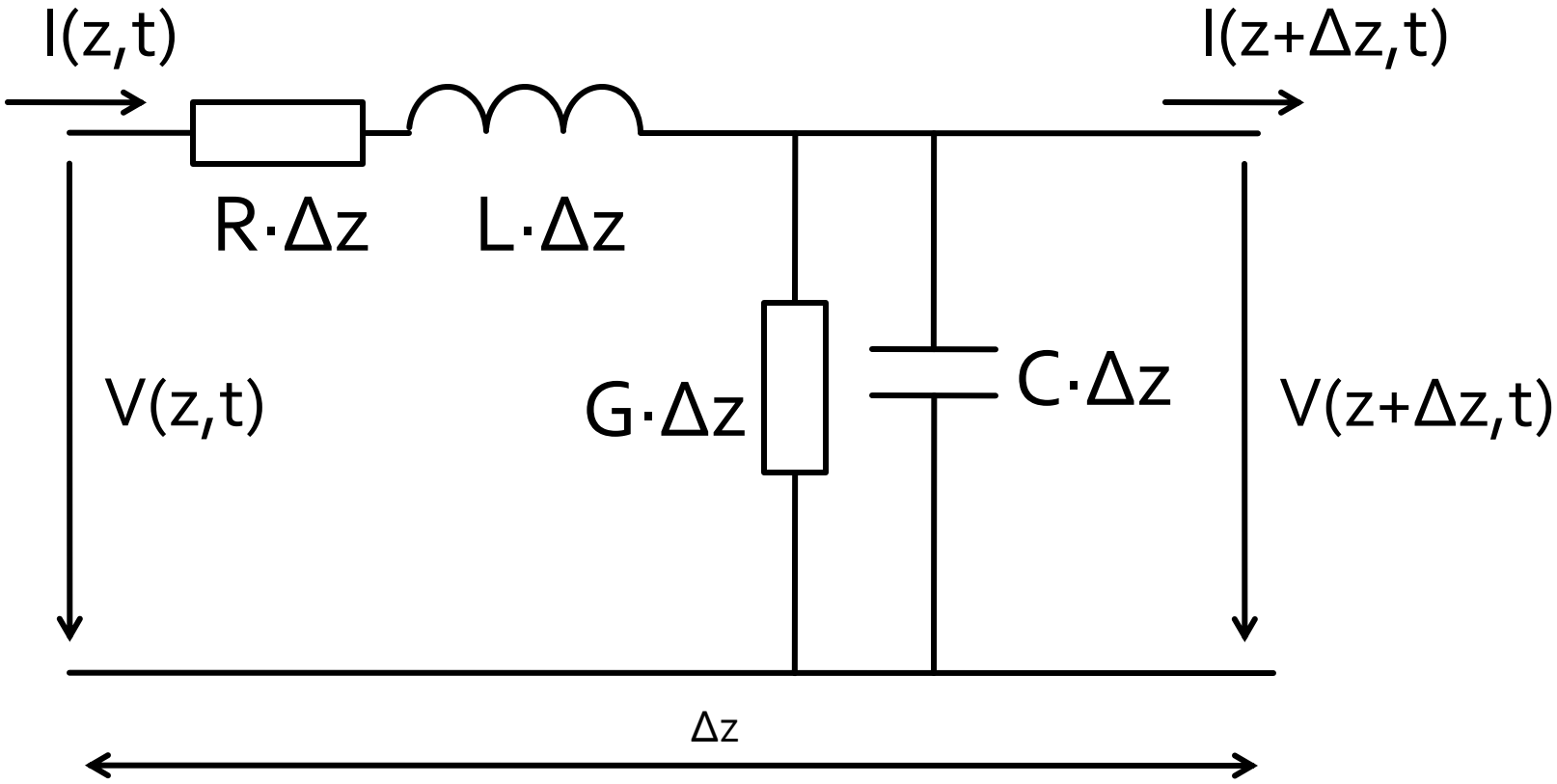
- Comportarea (descrierea) unui circuit depinde de lungimea sa electrica la frecventele de interes
  - E≈o → Kirchhoff
  - E>o → propagare

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left(\frac{l}{\lambda} \right)$$



Linie de transmisie model echivalent

- mod TEM, doi conductori, parametri **lineici**



- parametri **lineici, distribuiti** R, L, G, C (Ω/m, H/m ...)

Ecuatiile telegrafistilor

- domeniu timp

$$\frac{\partial v(z,t)}{\partial z} = -R \cdot i(z,t) - L \cdot \frac{\partial i(z,t)}{\partial t} \qquad \text{K II}$$

$$\frac{\partial i(z,t)}{\partial z} = -G \cdot v(z,t) - C \cdot \frac{\partial v(z,t)}{\partial t} \qquad \text{K I}$$

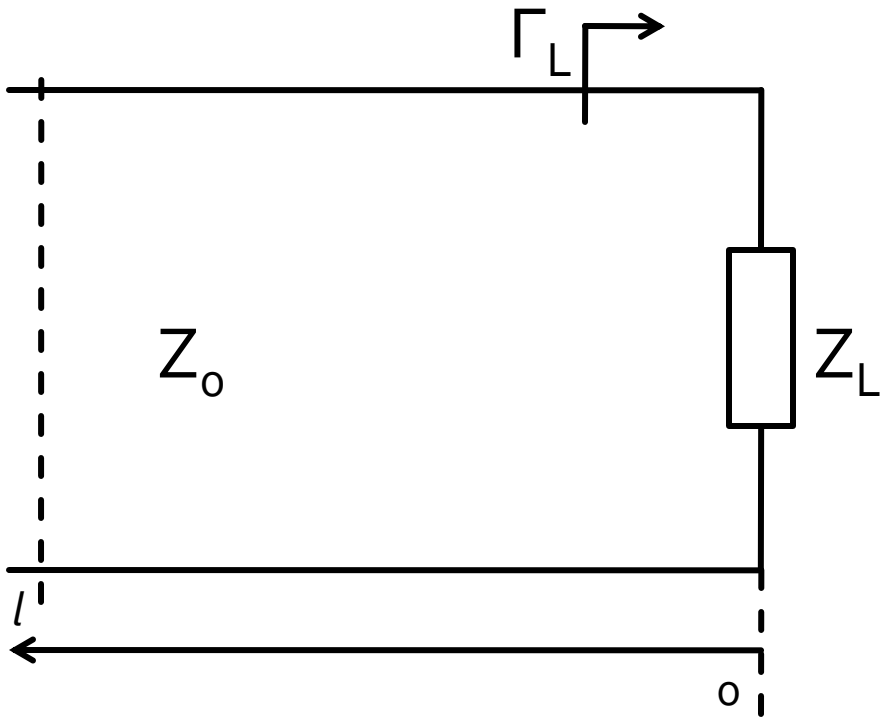
- semnale sinusoidale

$$\frac{dV(z)}{dz} = -(R + j \cdot \omega \cdot L) \cdot I(z)$$

$$\frac{dI(z)}{dz} = -(G + j \cdot \omega \cdot C) \cdot V(z)$$

$$\Big/ \frac{d}{dz}(\ldots)$$

Linie fara pierderi



$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\cdot\beta\cdot z} - \frac{V_0^-}{Z_0} e^{j\cdot\beta\cdot z}$$

$$Z_L = \frac{V(0)}{I(0)} \qquad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\cdot Z_0$$

- coeficient de reflexie in tensiune

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

  - Z_o real

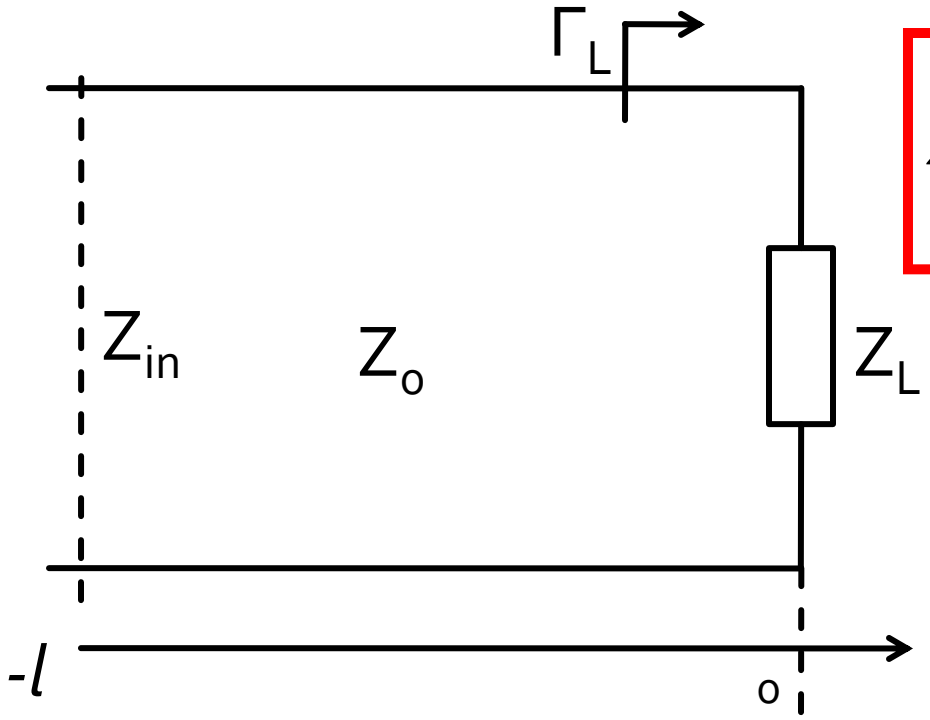
Linie fara pierderi

$$P_{avg} = \frac{1}{2} \cdot \frac{\left|V_0^+\right|^2}{Z_0} \cdot \left(1 - \left|\Gamma\right|^2\right)$$

- Puterea medie e constanta in lungul liniei
  - (**fara** P_{avg}(~~≠~~))
  - poate fi masurata
- Se utilizeaza puterea pentru a caracteriza amplitudinea semnalului
  - un punct de vedere "energetic" (fizica)
  - energie mai mare = semnal "mai mare"

Linie fara pierderi

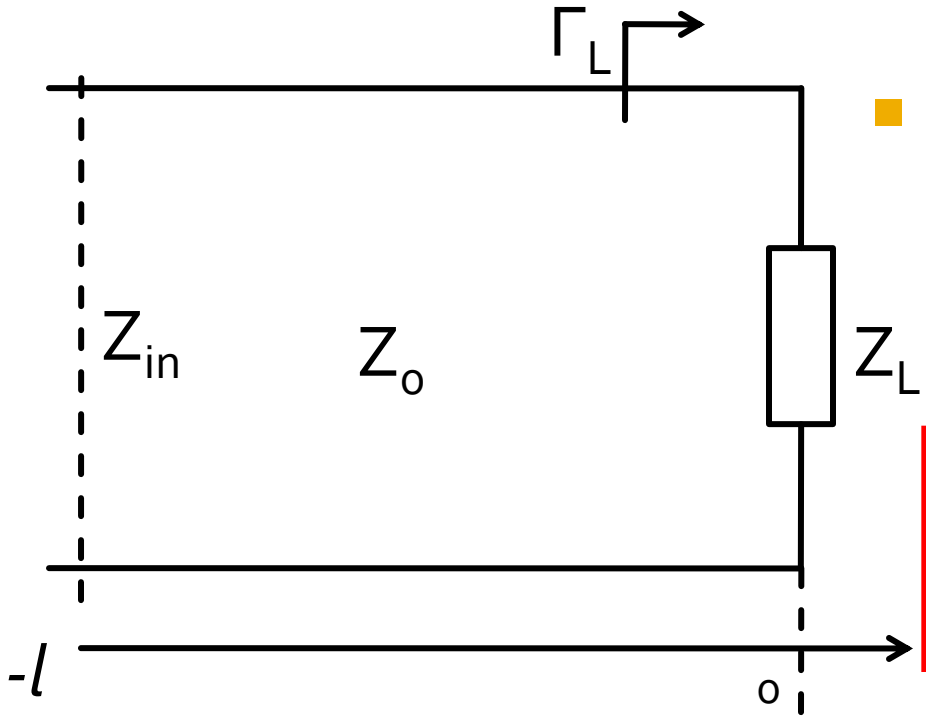
- impedanta la intrarea liniei de impedanta caracteristica Z_o, de lungime l, terminata cu impedanta Z_L



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

Linie fara pierderi, cazuri particulare

- $l = k \cdot \lambda/2$ $\quad \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = k \cdot \pi \quad \tan \beta \cdot l = 0$ $\quad Z_{in} = Z_L$
- $l = \lambda/4 + k \cdot \lambda/2$ $\quad \tan \beta \cdot l \to \infty$ $\quad Z_{in} = \frac{Z_0^2}{Z_L}$



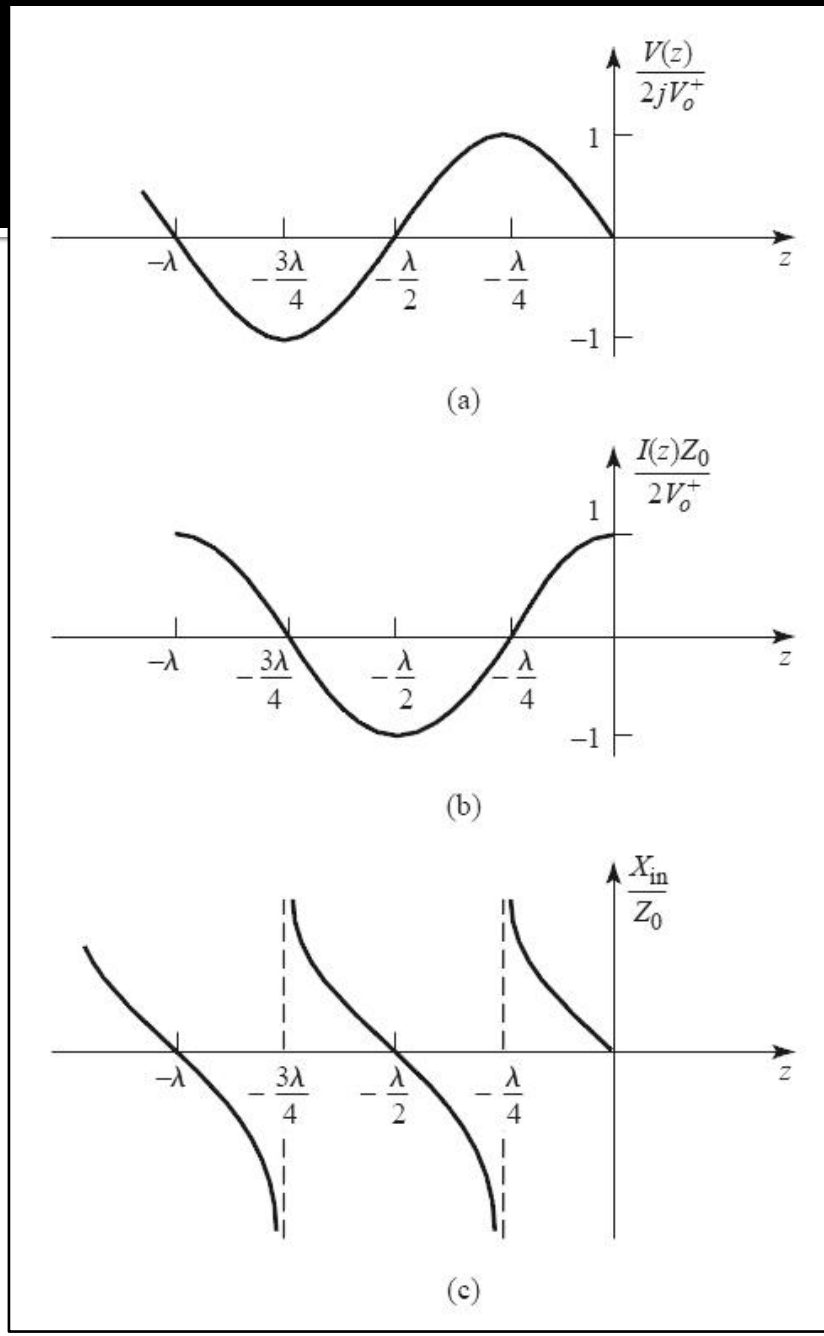
- Transformatorul in sfert de lungime de unda

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

Linie in scurtcircuit

- Z_L = o
- reactanța pură
  - +/- → in funcție de *l*

$$Z_{in} = j \cdot Z_0 \cdot \tan \beta \cdot l$$



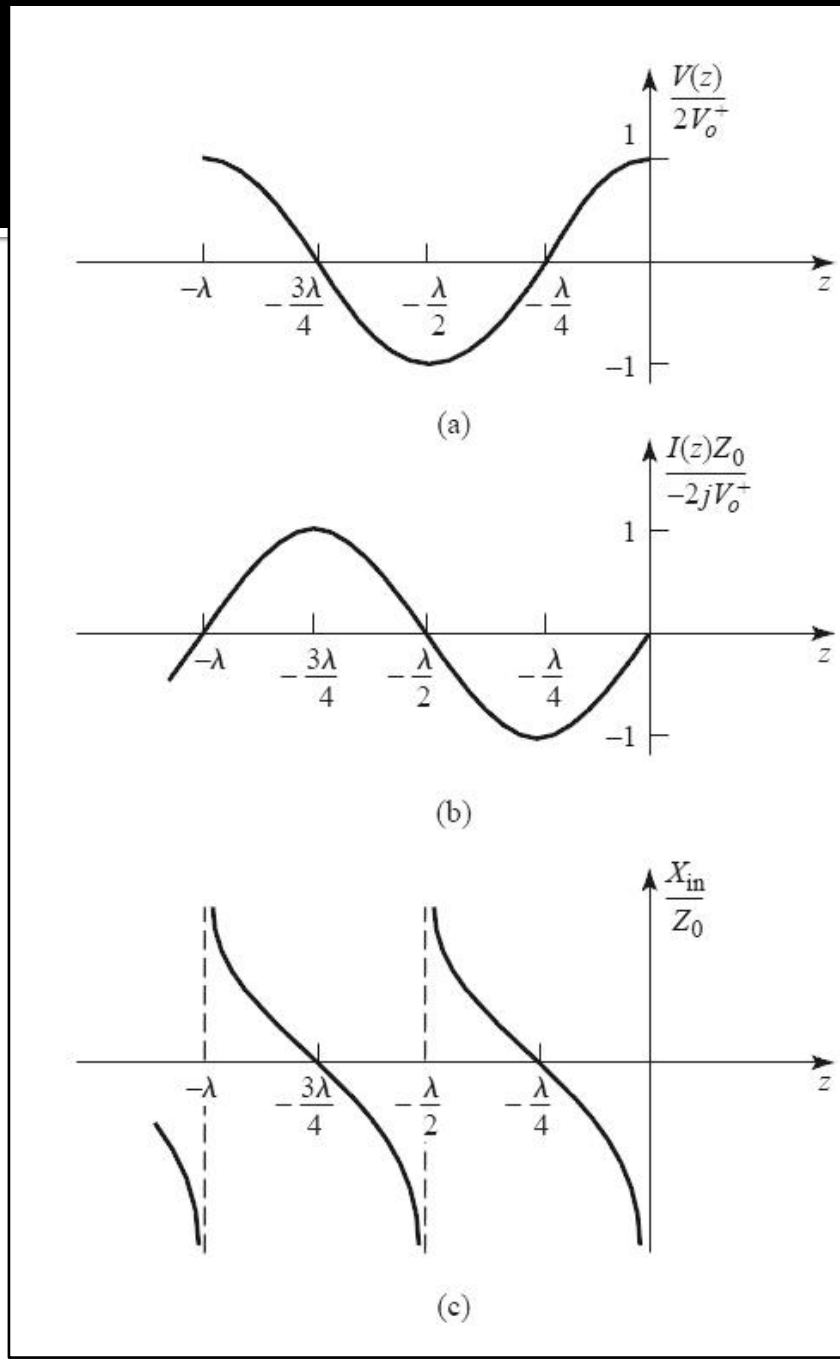
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

Linie in gol

- $Z_L = \infty \longrightarrow 1 / Z_L = 0$
- reactanța pură
  - +/- → in funcție de *l*

$$Z_{in} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

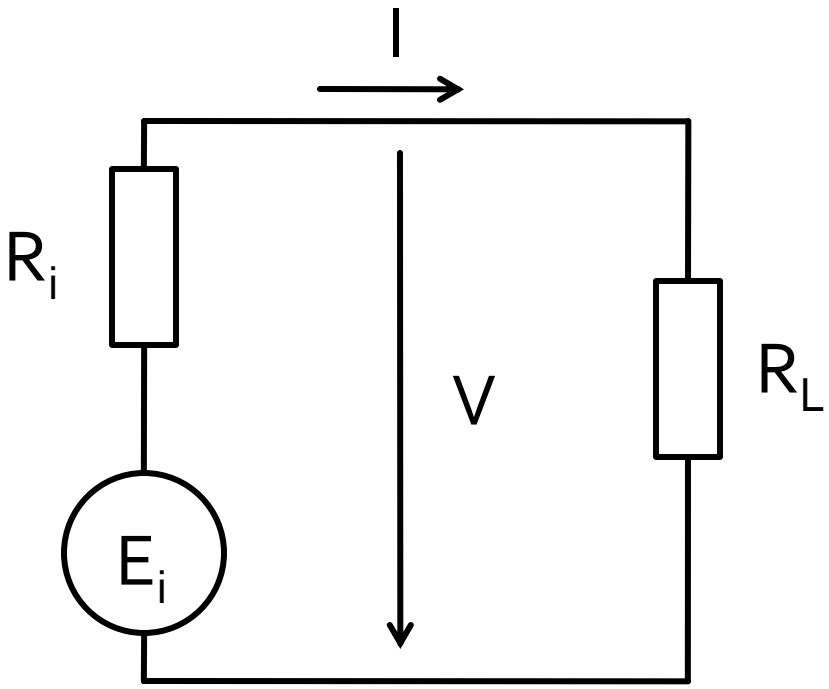


Transfer de putere

Adaptarea de impedanța

Adaptare, impedante reale

- Generator adaptat la sarcina



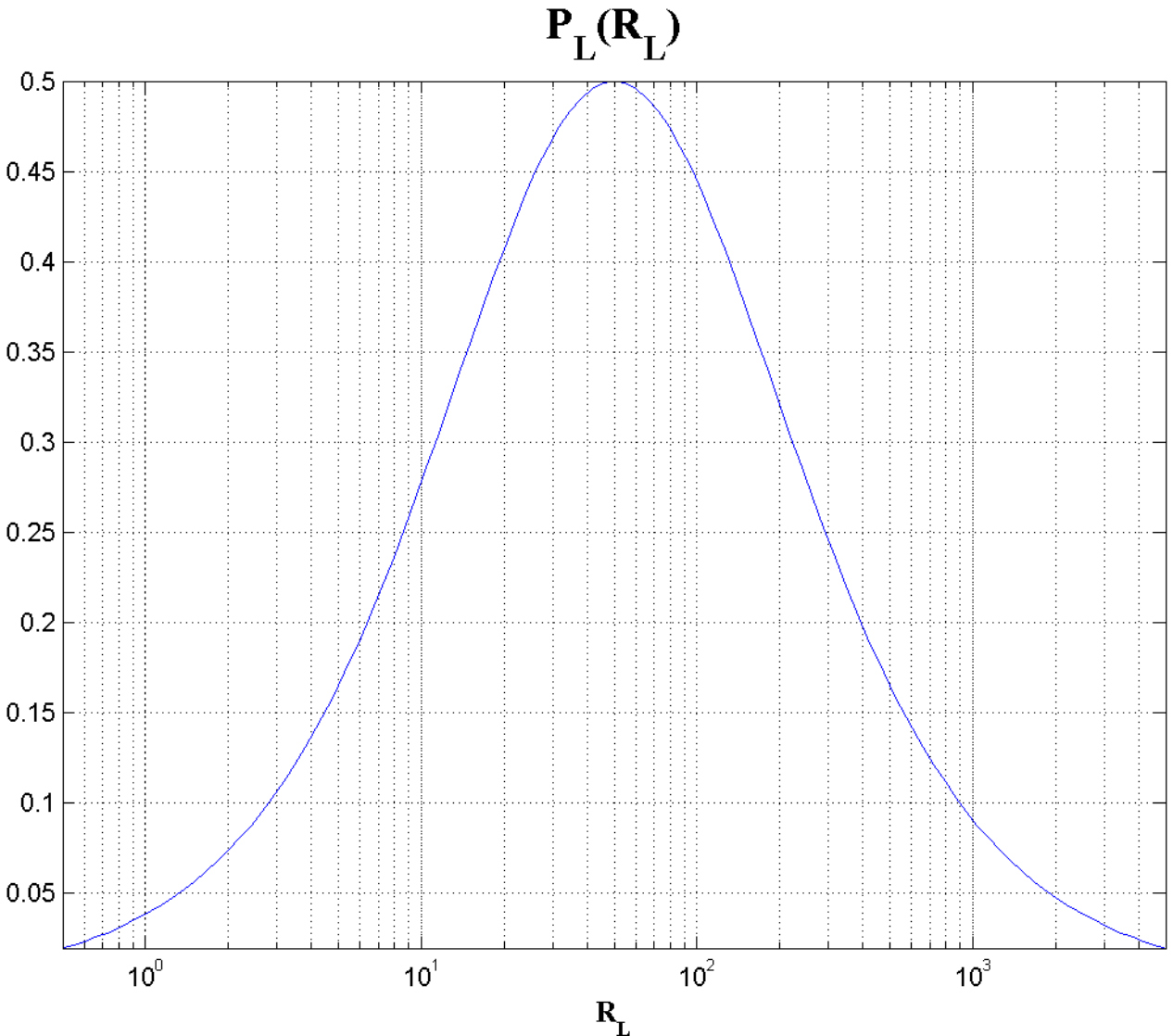
$$I = \frac{E_i}{R_i + R_L}$$

$$V = \frac{E_i \cdot R_L}{R_i + R_L}$$

$$P_L = R_L \cdot I^2$$

$$P_L = \frac{R_L \cdot E_i^2}{(R_i + R_L)^2}$$

Adaptare , impedante reale



Adaptare dpdv al puterii

$$R_i > 0, R_L > 0$$

$$P_L = \frac{|E_i|^2}{4R_i + \frac{(R_i - R_L)^2}{R_L} + \frac{(X_i + X_L)^2}{R_L}}$$

$$P_{L\max} = \frac{|E_i|^2}{4R_i} \equiv P_a$$

$$R_L = R_i, X_L = -X_i$$

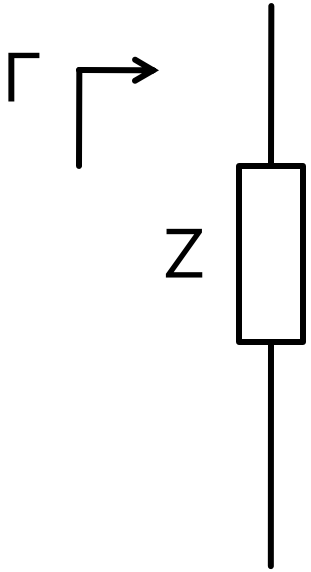
- P_a : Puterea disponibila (**available**)

$$Z_L = Z_i^*$$

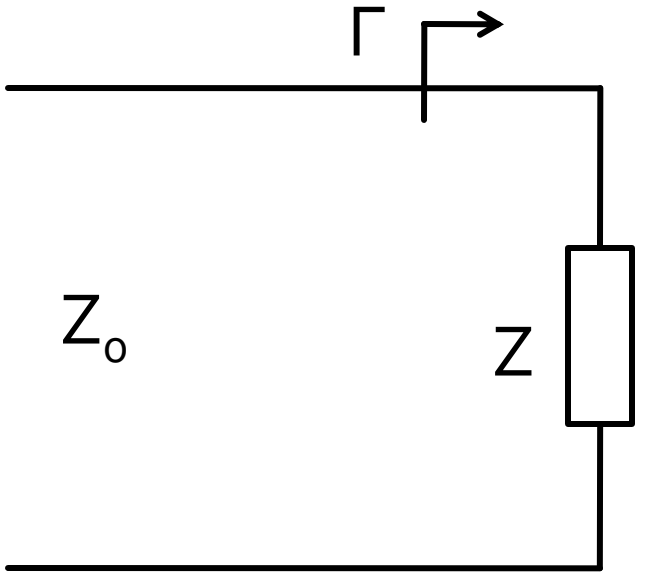
Continuare

Coeficient de reflexie

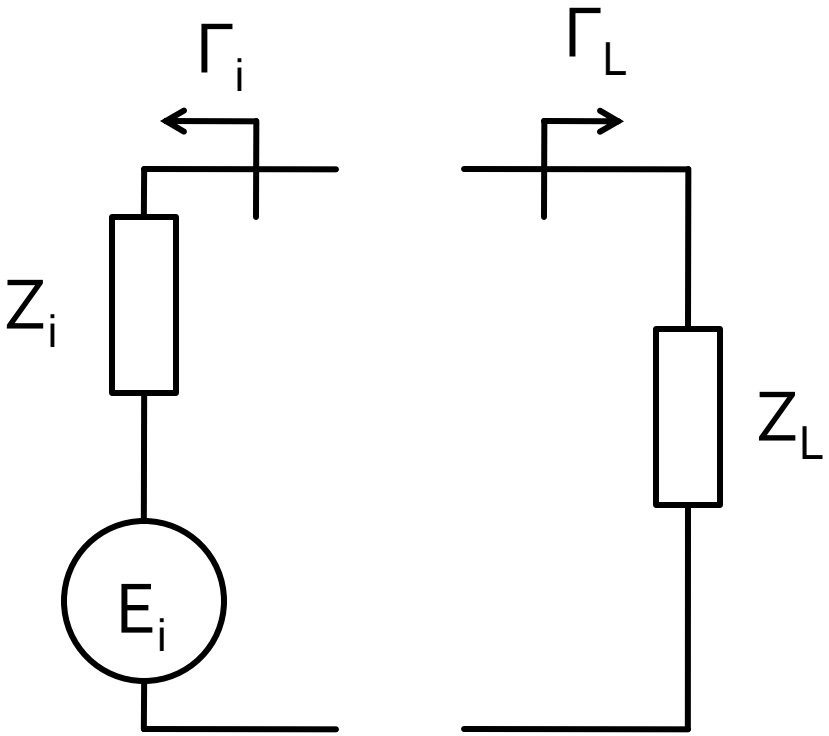
- Un Z_o oarecare ales ca referinta



$$\Gamma = \frac{Z - Z_0^*}{Z + Z_0}$$



Adaptare dpdv al puterii



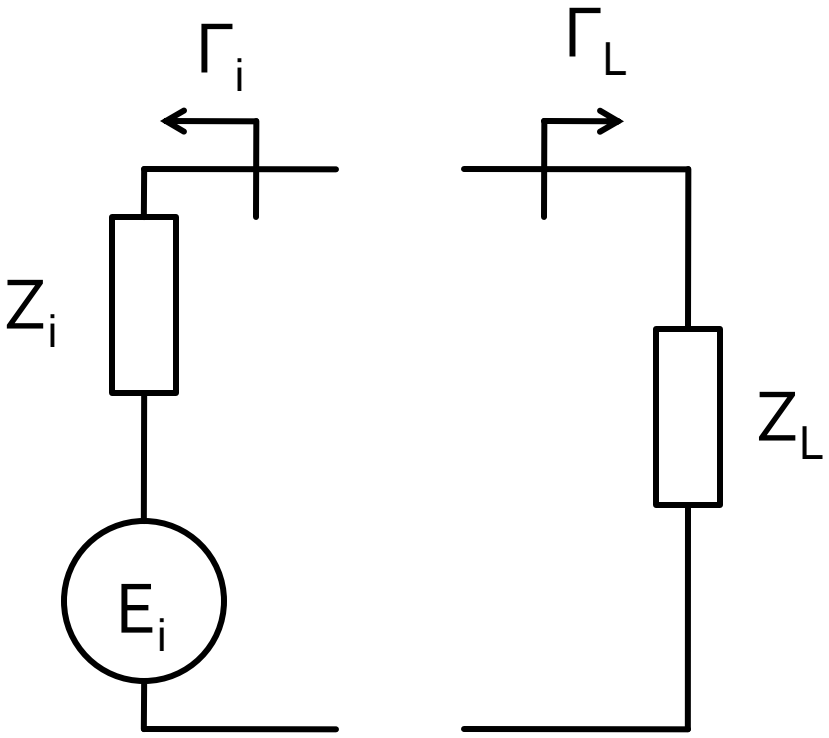
$$\Gamma_i = \frac{Z_i - Z_0^*}{Z_i + Z_0}$$

$$\Gamma_i = \frac{(R_i - R_0) + j \cdot (X_i + X_0)}{(R_i + R_0) + j \cdot (X_i + X_0)}$$

$$\Gamma_L = \frac{Z_L - Z_0^*}{Z_L + Z_0}$$

$$\Gamma_L = \frac{(R_L - R_0) + j \cdot (X_L + X_0)}{(R_L + R_0) + j \cdot (X_L + X_0)}$$

Adaptare dpdv al puterii



$$\Gamma_i = \frac{Z_i - Z_0^*}{Z_i + Z_0} = 1 - \frac{Z_0 + Z_0^*}{Z_i + Z_0}$$

$$\Gamma_L = \frac{Z_L - Z_0^*}{Z_L + Z_0} = 1 - \frac{Z_0 + Z_0^*}{Z_L + Z_0}$$

Adaptare: $Z_L = Z_i^*$

$$\Gamma_i^* = 1 - \frac{Z_0^* + Z_0}{Z_i^* + Z_0^*} = 1 - \frac{Z_0^* + Z_0}{Z_L + Z_0^*}$$

Daca se alege un Zo real $\Gamma_L = \Gamma_i^*$

Adaptare dpdv al puterii

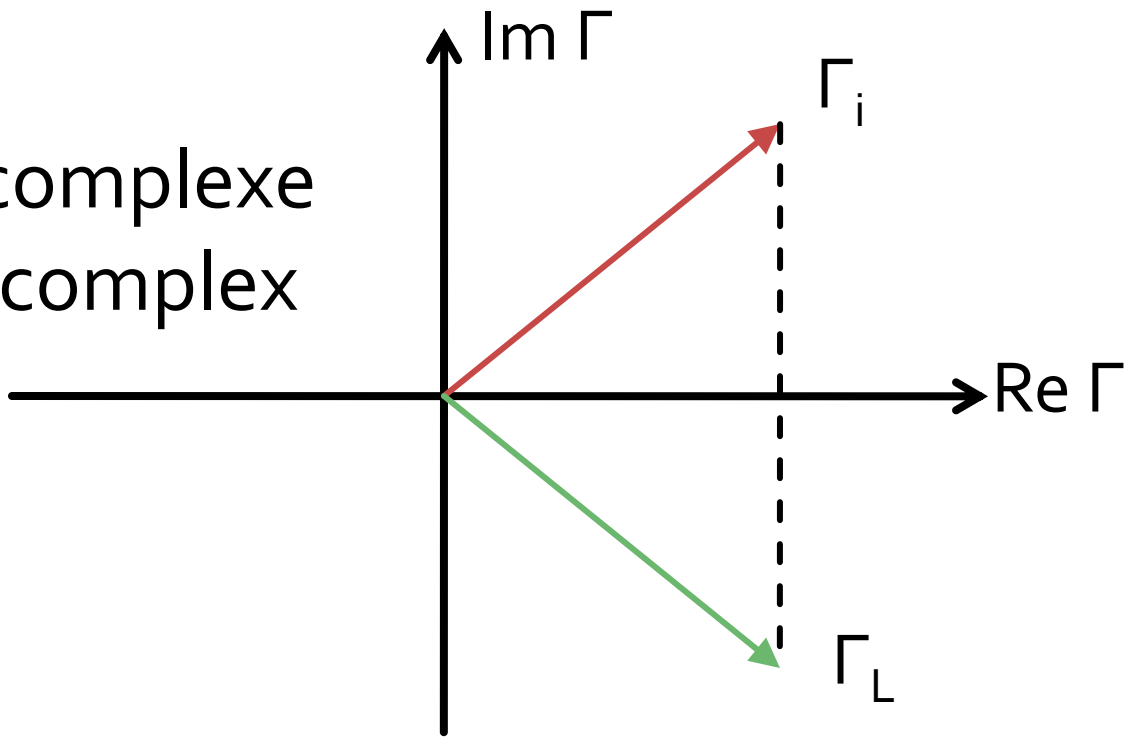
$$Z_L = Z_i^*$$

Daca se alege un Zo real

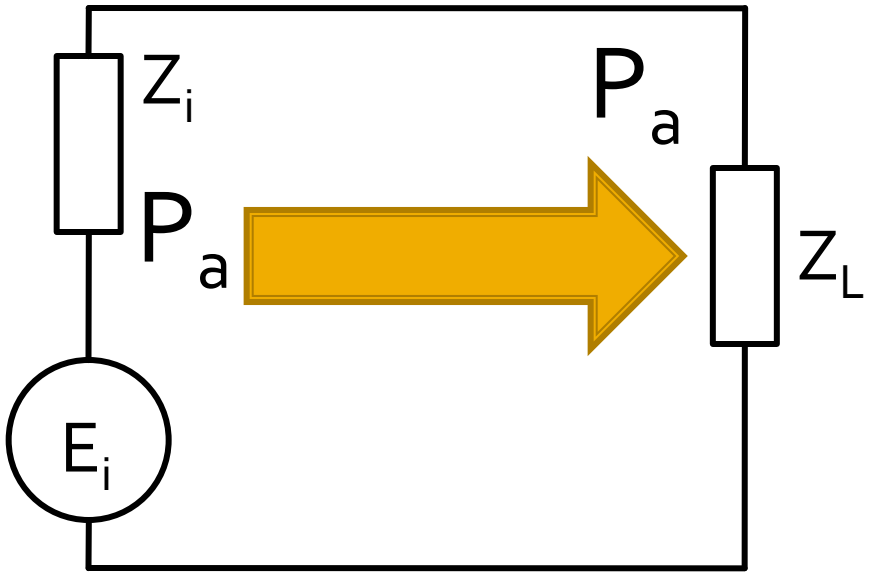
$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

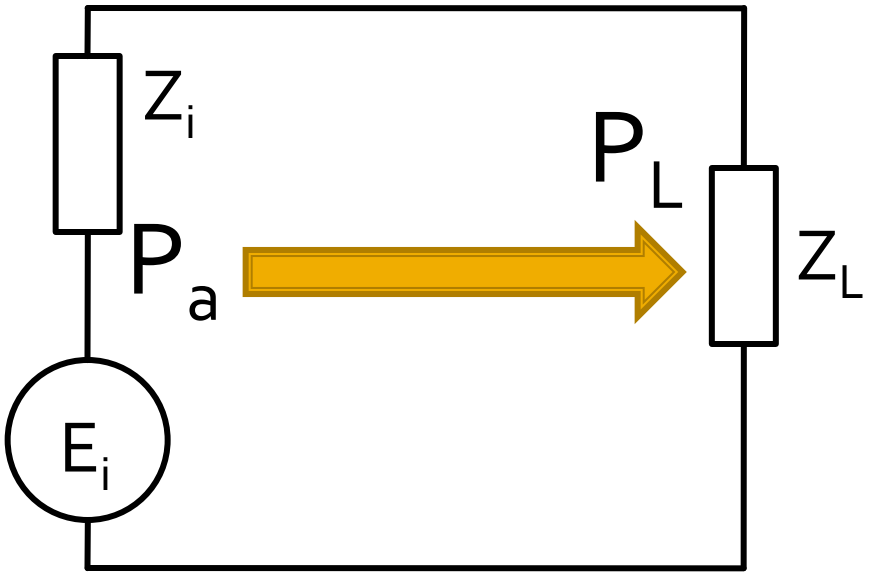
$$\Gamma_L = \Gamma_i^*$$

- numere complexe
- in planul complex



Reflexie de putere / Model





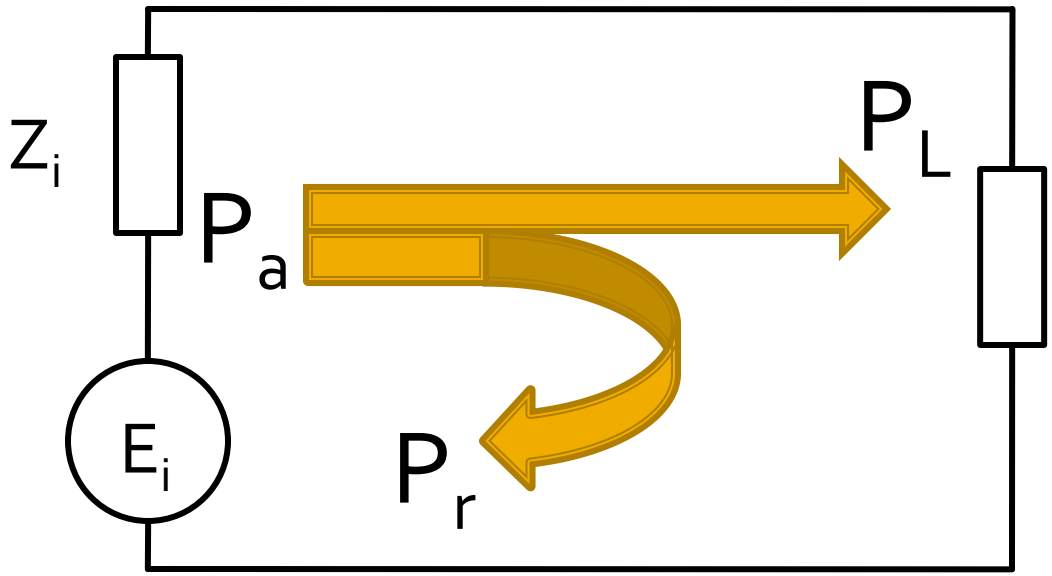
$$Z_L = Z_i^*$$

$$P_L = P_a$$

$$Z_L \neq Z_i^*$$

$$P_L < P_a$$

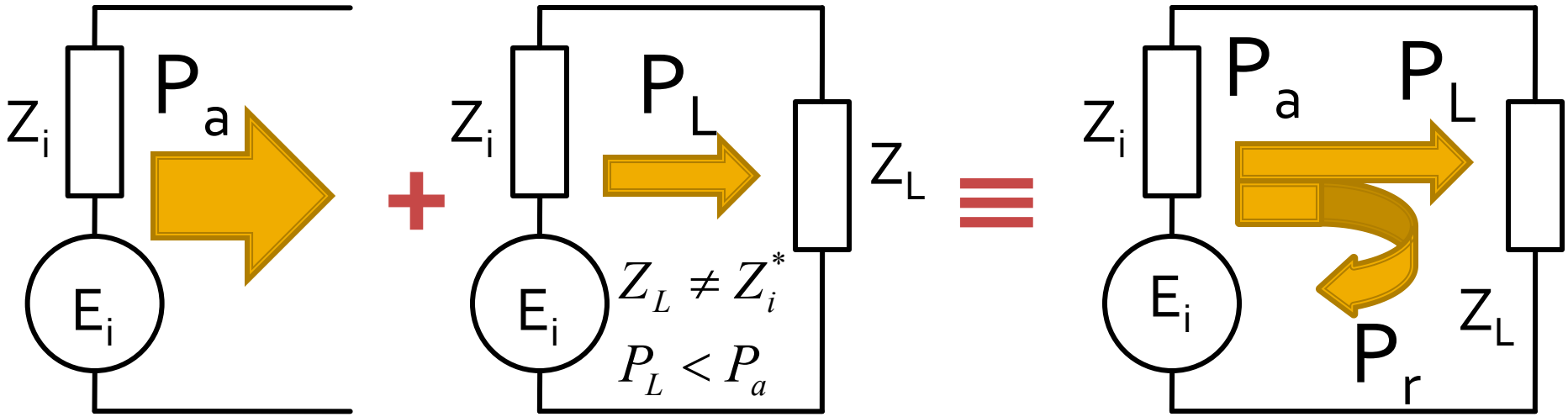
Reflexie de putere / Model



$$Z_L \neq Z_i^*$$

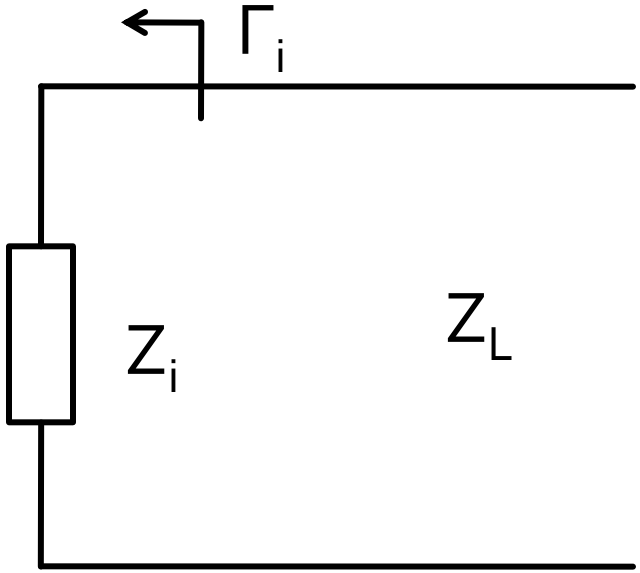
- ~~Putere reflectata~~
- Putere a undei reflectate

Reflexie de putere / Model

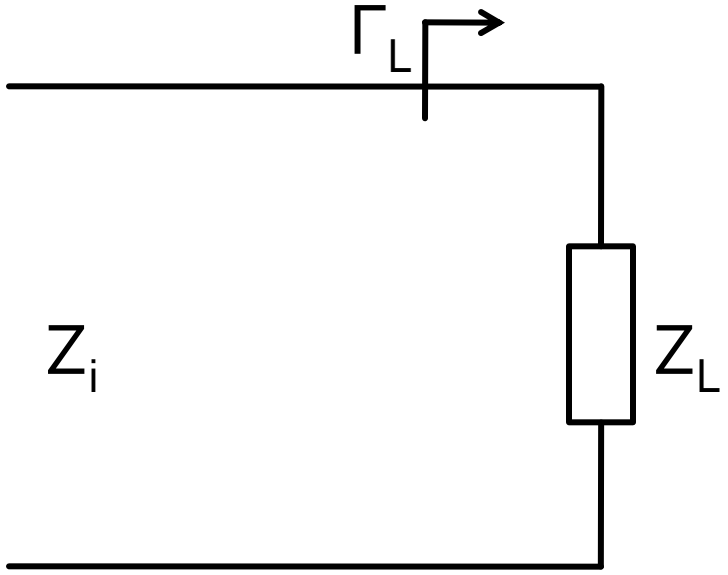


- Generatorul are posibilitatea de a oferi o anumita putere maxima de semnal P_a
- Pentru o sarcina oarecare, acesteia i se ofera o putere de semnal mai mica $P_L < P_a$
- Se intampla **"ca si cum"** (model) o parte din putere se reflecta $P_r = P_a - P_L$
- Puterea este o marime **scalara!**

Coeficienti de reflexie



$$\Gamma_i = \frac{Z_i - Z_L^*}{Z_i + Z_L}$$



$$\Gamma_L = \frac{Z_L - Z_i^*}{Z_L + Z_i}$$

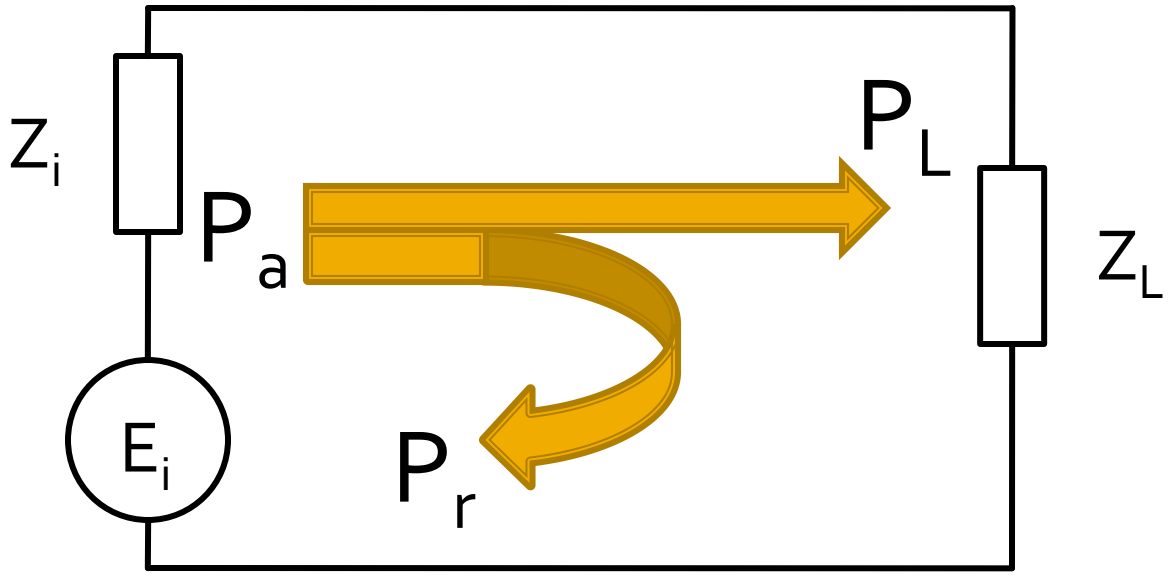
Coeficienti de reflexie

$$\Gamma_i = \frac{(R_i - R_L) + j \cdot (X_i + X_L)}{(R_i + R_L) + j \cdot (X_i + X_L)} \qquad \Gamma_L = \frac{(R_L - R_i) + j \cdot (X_L + X_i)}{(R_L + R_i) + j \cdot (X_L + X_i)}$$

$$|\Gamma_i| = \frac{|(R_i - R_L) + j \cdot (X_i + X_L)|}{|(R_i + R_L) + j \cdot (X_i + X_L)|} = \frac{\sqrt{(R_i - R_L)^2 + (X_i + X_L)^2}}{\sqrt{(R_i + R_L)^2 + (X_i + X_L)^2}} = |\Gamma_L|$$

$$|\Gamma_i| = |\Gamma_L| \equiv |\Gamma|$$

Reflexie de putere / Model



$$P_a = \frac{|E_i|^2}{4R_i}$$

$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i+R_L)^2 + (X_i+X_L)^2}$$

$$P_r = P_a - P_L = \frac{|E_i|^2}{4R_i} - \frac{R_L \cdot |E_i|^2}{(R_i+R_L)^2 + (X_i+X_L)^2} = \frac{|E_i|^2}{4R_i} \cdot \left[1 - \frac{4R_L \cdot R_i}{(R_i+R_L)^2 + (X_i+X_L)^2}\right]$$

$$P_r = \frac{|E_i|^2}{4R_i} \cdot \left[\frac{(R_i-R_L)^2 + (X_i+X_L)^2}{(R_i+R_L)^2 + (X_i+X_L)^2}\right] = P_a \cdot |\Gamma|^2$$

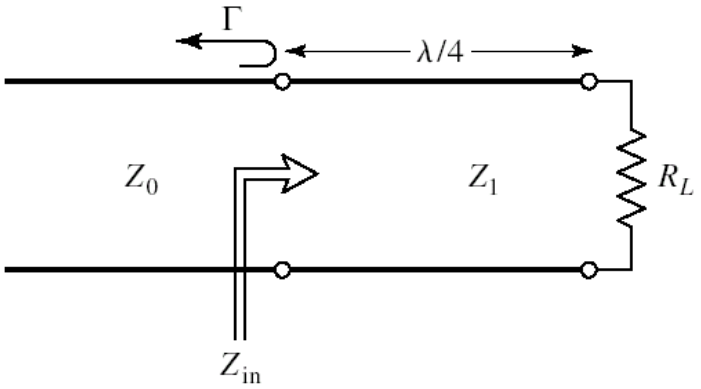
- $|\Gamma|^2$ **este** un coeficient de reflexie in putere

Transformatorul in sfert de lungime de unda

Adaptarea de impedanța

Transformatorul in sfert de lungime de unda

- Feed line – linie de intrare cu impedanta caracteristica Z_0
- Sarcina cu impedanta (**rezistiva!**) R_L
- Dorim adaptarea sarcinei la fider cu o linie de lungime $\lambda/4$ si impedanta caracteristica Z_1

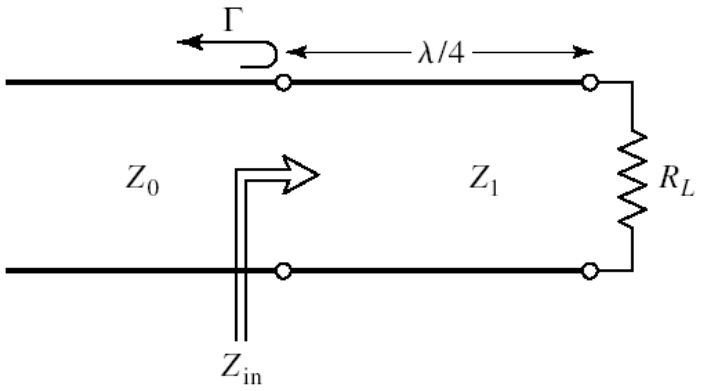


$$Z_{in} = Z_1 \frac{1+\Gamma e^{-2j\beta l}}{1-\Gamma e^{-2j\beta l}}$$

$$\Gamma_O = \frac{V_0^-}{V_0^+} = \frac{R_L - Z_1}{R_L + Z_1}$$

$$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan(\beta l)}{Z_1 + jR_L \tan(\beta l)}$$

Transformatorul in sfert de lungime de unda



$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2} \qquad Z_{in} = \frac{Z_1^2}{R_L}$$

$$\Gamma_{in} = \frac{Z_1^2 - Z_0 \cdot R_L}{Z_1^2 + Z_0 \cdot R_L} \qquad \Gamma_{in} = 0 \Rightarrow Z_1 = \sqrt{Z_0 R_L}$$

- Pe fider (Z_0) avem doar unda progresiva
- Pe linia in sfert de lungime de unda (Z_1) avem unda stationara

Transformatorul in sfert de lungime de unda

- Punct de vedere fizic

$$\Gamma = \Gamma_1 - T_1 T_2 \Gamma_3 + T_1 T_2 \Gamma_2 \Gamma_3^2 - T_1 T_2 \Gamma_2^2 \Gamma_3^3 + \cdots$$

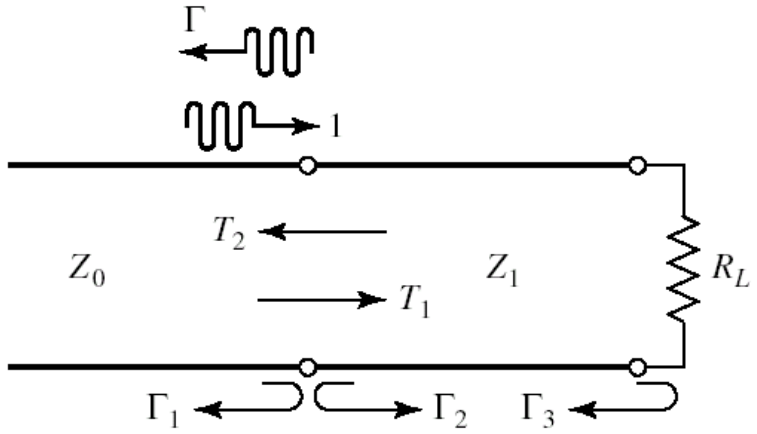
$$= \Gamma_1 - T_1 T_2 \Gamma_3 \sum_{n=0}^{\infty} (-\Gamma_2 \Gamma_3)^n.$$

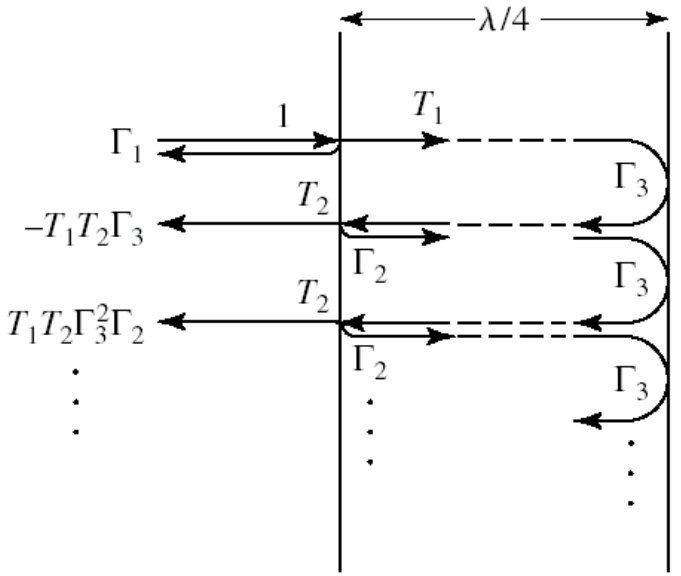
$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0},$$

$$\Gamma_2 = \frac{Z_0 - Z_1}{Z_0 + Z_1} = -\Gamma_1,$$

$$\Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1},$$

$$\left.\begin{aligned} T_1 &= \frac{2Z_1}{Z_1 + Z_0}, \\ T_2 &= \frac{2Z_0}{Z_1 + Z_0}. \end{aligned}\right\} \quad T = 1 - \Gamma$$



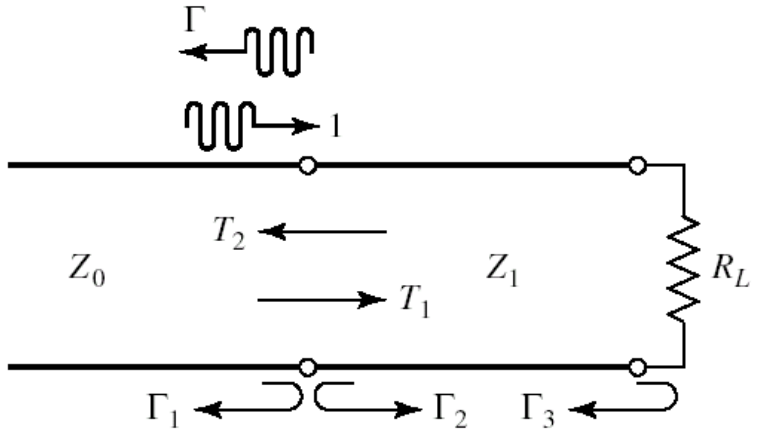


Transformatorul in sfert de lungime de unda

- Punct de vedere fizic

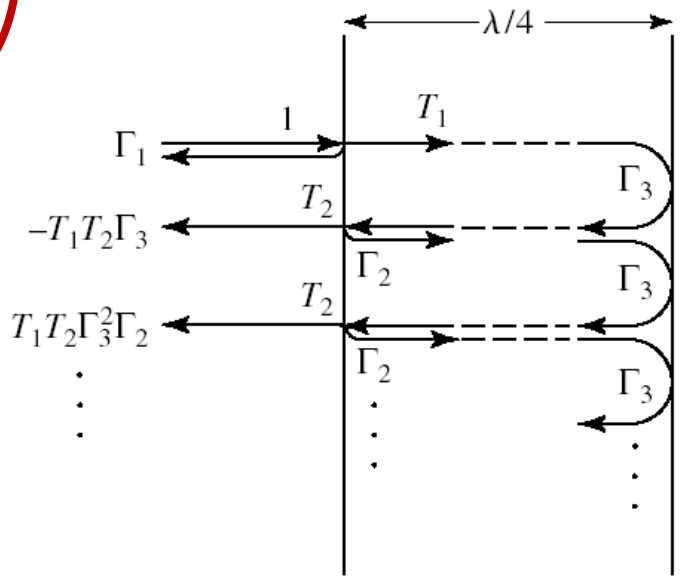
$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \qquad \text{for } |x| < 1,$$

$$\Gamma = \Gamma_1 - \frac{T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3} = \frac{\Gamma_1 + \Gamma_1 \Gamma_2 \Gamma_3 - T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3}.$$



$$\Gamma_1 - \Gamma_3\left(\Gamma_1^2 + T_1 T_2\right) = \frac{2\left(Z_1^2 - Z_0 R_L\right)}{(Z_1 + Z_0)(R_L + Z_1)},$$

$$\Gamma_1^2 + T_1 T_2 = \frac{(Z_1 - Z_0)^2}{(Z_1 + Z_0)^2} + \frac{4 Z_1 Z_0}{(Z_1 + Z_0)^2} = 1$$



$$\Gamma = 0 \quad \Leftrightarrow \quad Z_1^2 - Z_0 \cdot R_L = 0$$

Caracteristica de frecventa

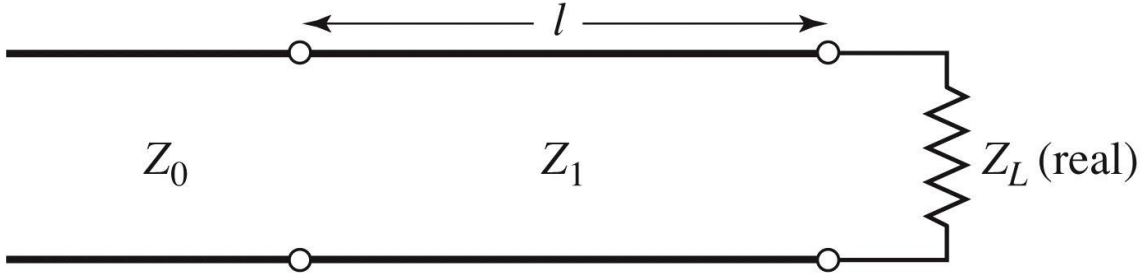


Figure 5.10
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$$Z_1 = \sqrt{Z_0 \cdot Z_L}$$

- (**doar**) la frecventa f_o

$$l = \frac{\lambda_0}{4} \qquad \beta_0 \cdot l = \frac{2\pi}{\lambda_0} \cdot \frac{\lambda_0}{4} = \frac{\pi}{2}$$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta \cdot l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

$$\theta \overset{not}{=} \beta \cdot l \qquad t \overset{not}{=} \tan(\beta \cdot l)$$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot t}{Z_1 + j \cdot Z_L \cdot t}$$

$$\Gamma = \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0} = \frac{Z_1(Z_L - Z_0) + jt\left(Z_1^2 - Z_0 Z_L\right)}{Z_1(Z_L + Z_0) + jt\left(Z_1^2 + Z_0 Z_L\right)}.$$

$$Z_1^2 = Z_0 \cdot Z_L$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0 + j2t\sqrt{Z_0 Z_L}}.$$

Caracteristica de frecventa

- calitatea adaptarii ≡coeficient de reflexie in putere

$$|\Gamma| = \frac{|Z_L - Z_0|}{\left[(Z_L + Z_0)^2 + 4t^2 Z_0 Z_L\right]^{1/2}}$$

$$= \frac{1}{\left\{(Z_L + Z_0)^2/(Z_L - Z_0)^2 + [4t^2 Z_0 Z_L/(Z_L - Z_0)^2]\right\}^{1/2}}$$

$$= \frac{1}{\left\{1 + [4Z_0 Z_L/(Z_L - Z_0)^2] + [4Z_0 Z_L t^2/(Z_L - Z_0)^2]\right\}^{1/2}}$$

$$= \frac{1}{\left\{1 + [4Z_0 Z_L/(Z_L - Z_0)^2]\ \sec^2\theta\right\}^{1/2}},$$

$$\sec\theta = \frac{1}{\cos\theta} \rightarrow$$

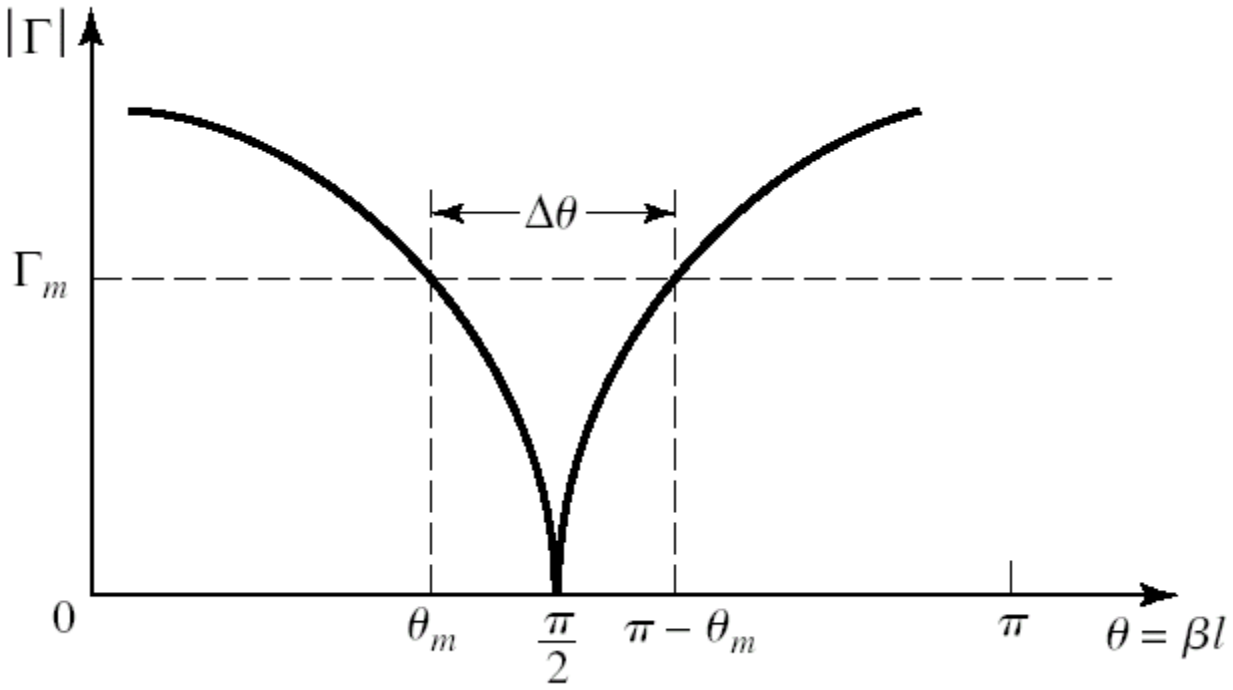
$$\sec^2\theta = 1 + \tan^2\theta = 1 + t^2$$

Caracteristica de frecventa

- ne intereseaza frecventa in jurul frecventei la care facem adaptarea (banda ingusta)

$$f \approx f_0 \qquad l \approx \frac{\lambda_0}{4} \qquad \theta \approx \frac{\pi}{2} \qquad \sec^2\theta = 1 + \tan^2\theta >> 1$$

$$|\Gamma| \cong \frac{|Z_L - Z_0|}{2 \cdot \sqrt{Z_0 \cdot Z_L}} \cdot |\cos\theta|$$



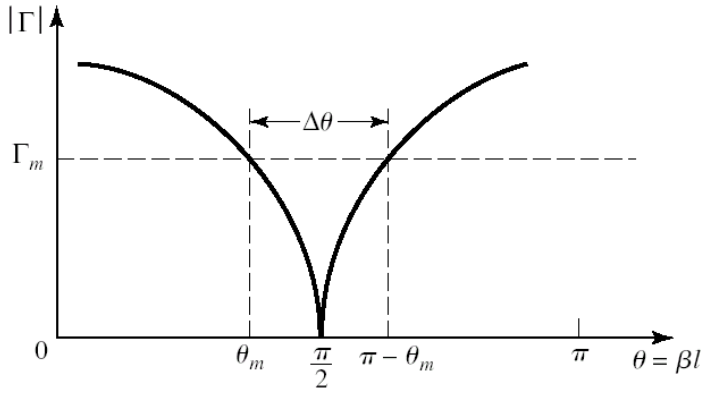
Caracteristica de frecventa

- Definim un maxim acceptat pentru coeficientul de reflexie Γ_m care va defini banda adaptarii, θ_m

$$\frac{1}{\Gamma_m^2} = 1 + \left(\frac{2\sqrt{Z_0 Z_L}}{Z_L - Z_0} \sec\theta_m \right)^2,$$

$$\cos\theta_m = \frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}.$$

$$\Delta\theta = 2\left(\frac{\pi}{2} - \theta_m\right)$$



- in linii TEM

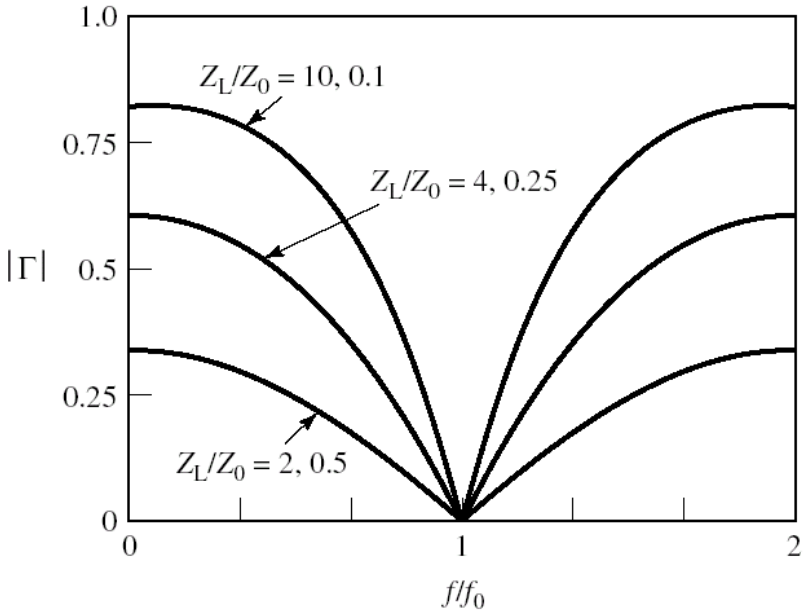
$$\theta = \beta \cdot l = \beta \cdot \frac{\lambda_0}{4} = \frac{2\pi \cdot f}{v_f} \cdot \frac{1}{4} \cdot \frac{v_f}{f_0} = \frac{\pi \cdot f}{2 f_0} \qquad f_m = \frac{2 \cdot \theta_m \cdot f_0}{\pi}$$

$$\frac{\Delta f}{f_0} = \frac{2 \cdot (f_0 - f_m)}{f_0} = 2 - \frac{4 \cdot \theta_m}{\pi} = 2 - \frac{4}{\pi} \cdot \cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}} \cdot \frac{2\sqrt{Z_0 \cdot Z_L}}{|Z_L - Z_0|} \right]$$

Caracteristica de frecventa

- Pentru linii non TEM constanta de propagare nu depinde liniar de frecventa, dar in practica influenta este minora in banda ingusta
- Sunt neglijate reactantele introduse de discontinuitati (Z_0 -> Z_1). Compensarea se face printr-o mica modificare a lungimii liniei
- Banda depinde de dezadaptarea initiala

cu cat dezadaptarea este mai mica
cu atat banda se obtine mai larga



Exemplu

- Transformator de adaptare cu o singura sectiune ($\lambda/4$) pentru a adapta o sarcina de 10Ω la o linie de 50 Ω la frecventa f_0=3GHz
  - banda pentru SWR<1.5

$$Z_1 = \sqrt{Z_0 Z_L} = \sqrt{(50)(10)} = 22.36\ \Omega, \qquad \Gamma_m = \frac{\text{SWR} - 1}{\text{SWR} + 1} = \frac{1.5 - 1}{1.5 + 1} = 0.2.$$

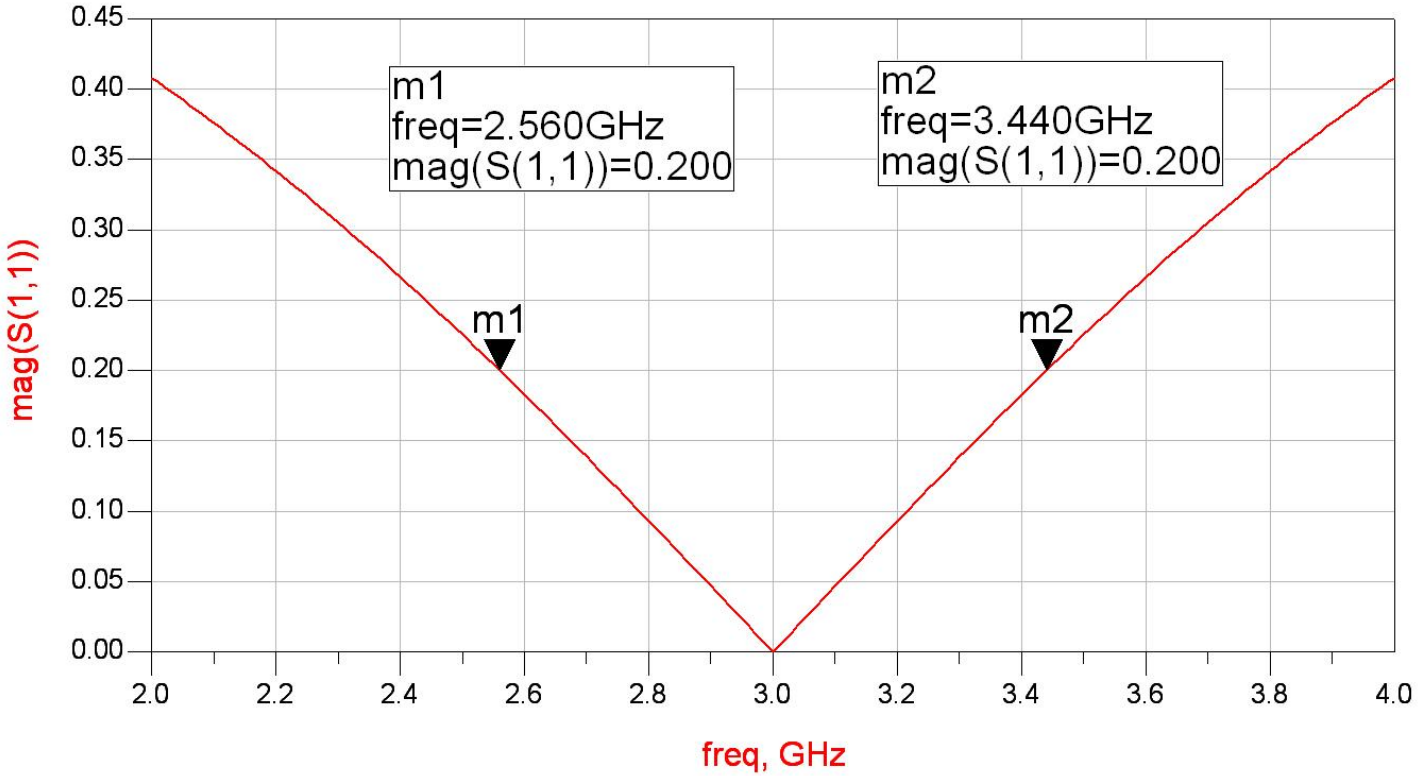
$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi} \cos^{-1} \left[\frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|} \right]$$

$$= 2 - \frac{4}{\pi} \cos^{-1} \left[\frac{0.2}{\sqrt{1 - (0.2)^2}} \frac{2\sqrt{(50)(10)}}{|10 - 50|} \right]$$

$$= 0.29, \text{ or } 29\%.$$

Simulare

- simulare ADS

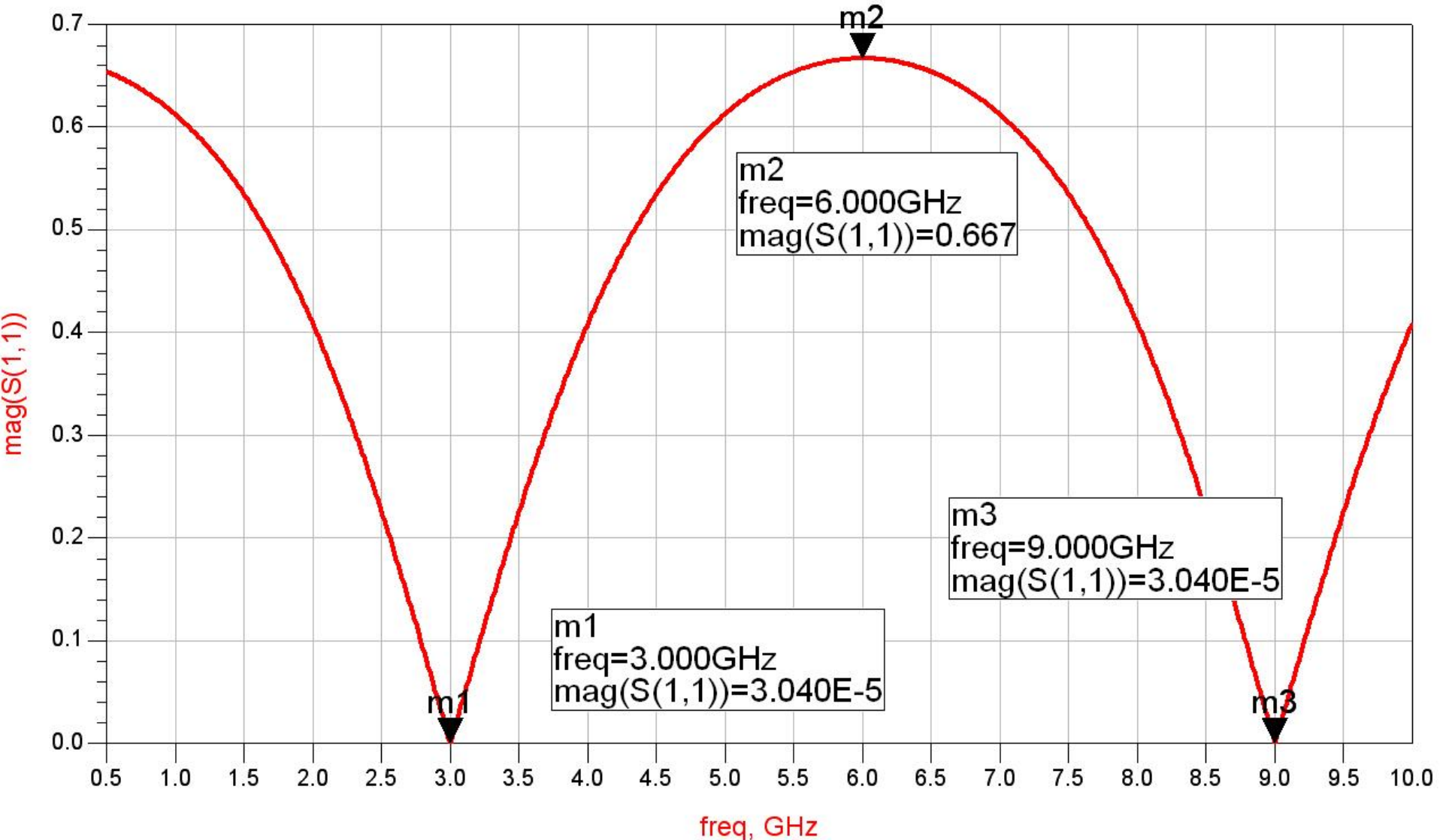


$$\Delta f = 0.88GHz$$

$$\left|\Gamma(3GHz)\right| = 3 \cdot 10^{-5}$$

$$\frac{\Delta f}{f_0} = \frac{0.88}{3} = 0.2933$$

Simulare banda larga



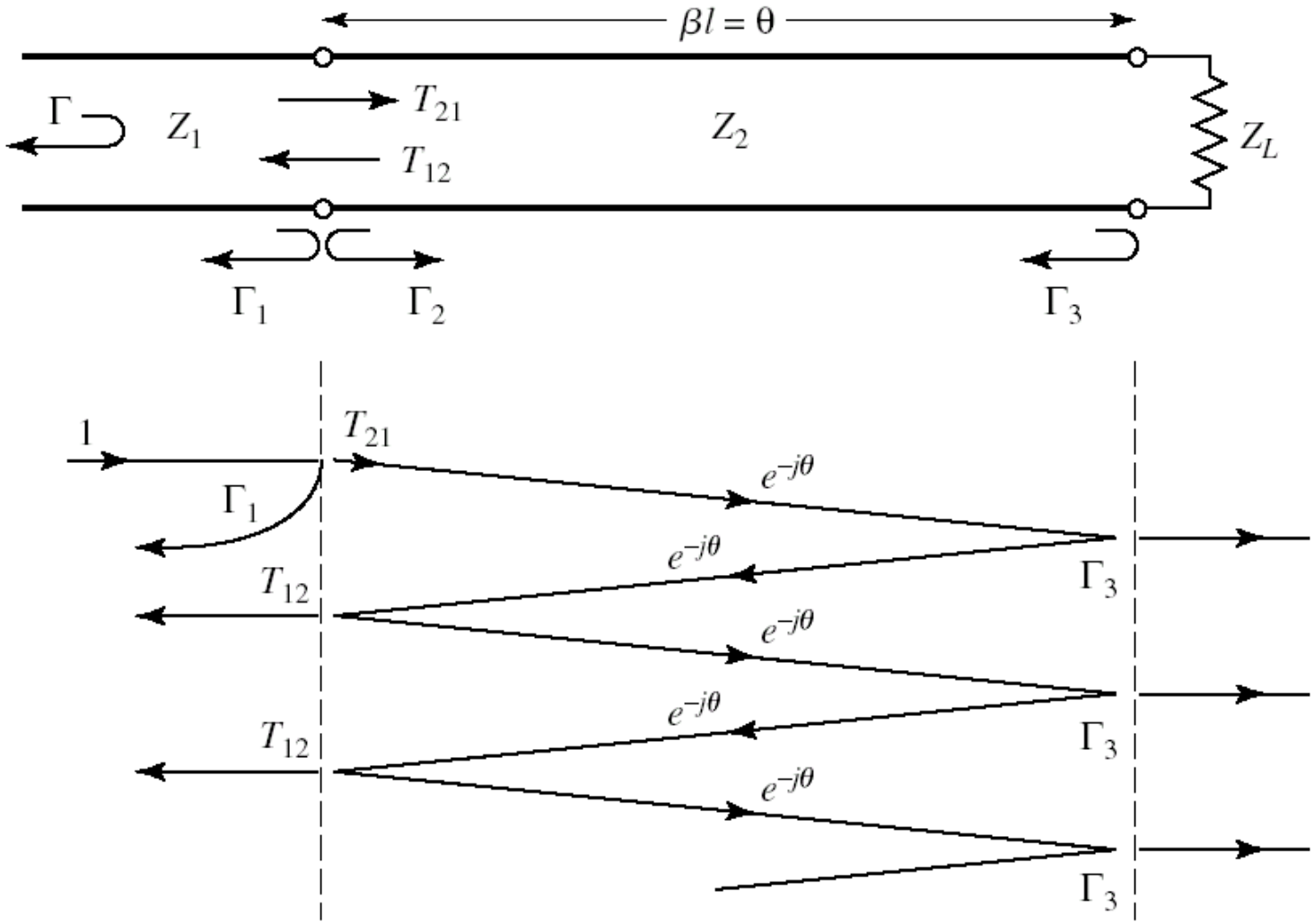
Adaptarea cu transformatoare de impedanta (Lab. 2)

Adaptarea de impedanța

Transformatoare de impedanta multisectiune

- Transformatorul in sfert de lungime de unda permite adaptarea oricarei impedante reale cu orice impedanta a fiderului (liniei).
- Daca banda necesara este mai mare decat cea oferita de transformatorul in sfert de lungime de unda se folosesc transformatoare multisectiune
  - caracteristica binomiala
  - tip Cebîşev

Teoria reflexiilor mici



Teoria reflexiilor mici

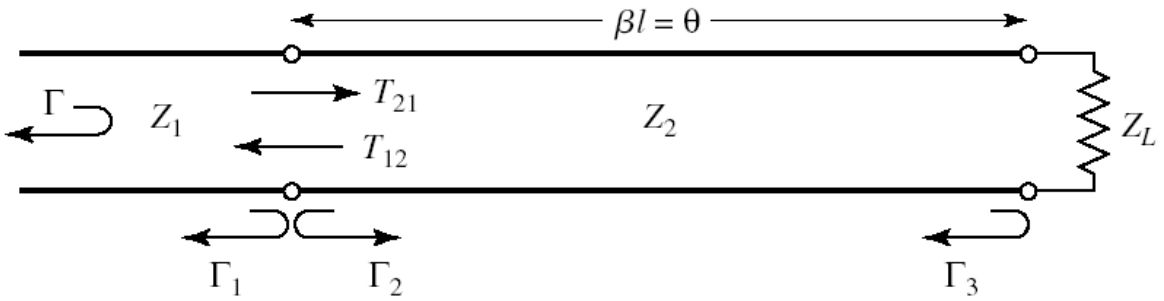
$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

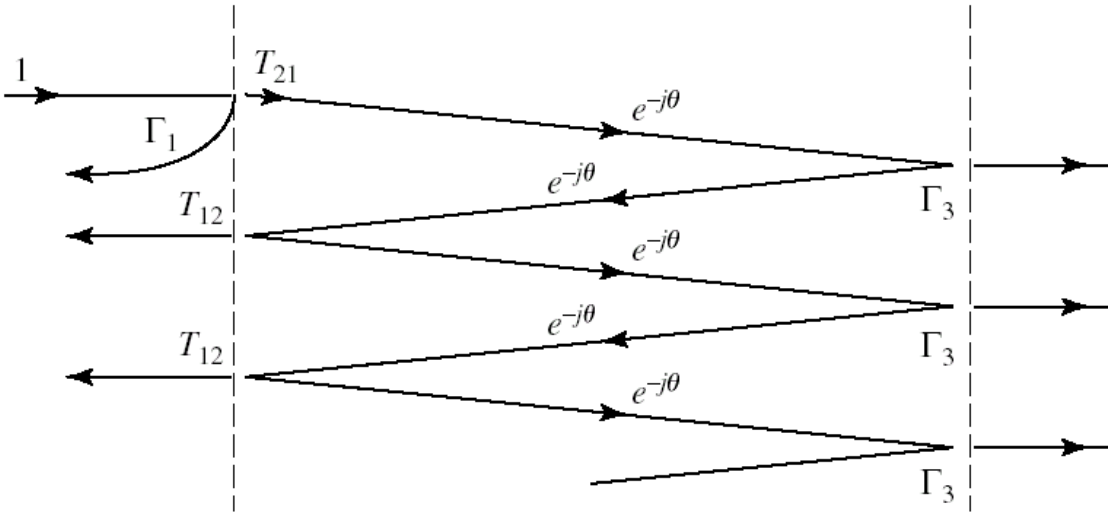
$$\Gamma_2 = -\Gamma_1$$

$$\Gamma_3 = \frac{Z_L - Z_2}{Z_L + Z_2}$$

$$T_{21} = 1 + \Gamma_1 = \frac{2 \cdot Z_2}{Z_1 + Z_2}$$

$$T_{12} = 1 + \Gamma_2 = \frac{2 \cdot Z_1}{Z_1 + Z_2}$$





$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} + T_{12} \cdot T_{21} \cdot \Gamma_3^2 \cdot \Gamma_2 \cdot e^{-4j\theta} + T_{12} \cdot T_{21} \cdot \Gamma_3^3 \cdot \Gamma_2^2 \cdot e^{-6j\theta} + \cdots$$

$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} \sum_{n=0}^{\infty} \Gamma_3^n \cdot \Gamma_2^n \cdot e^{-2jn\theta}$$

Teoria reflexiilor mici

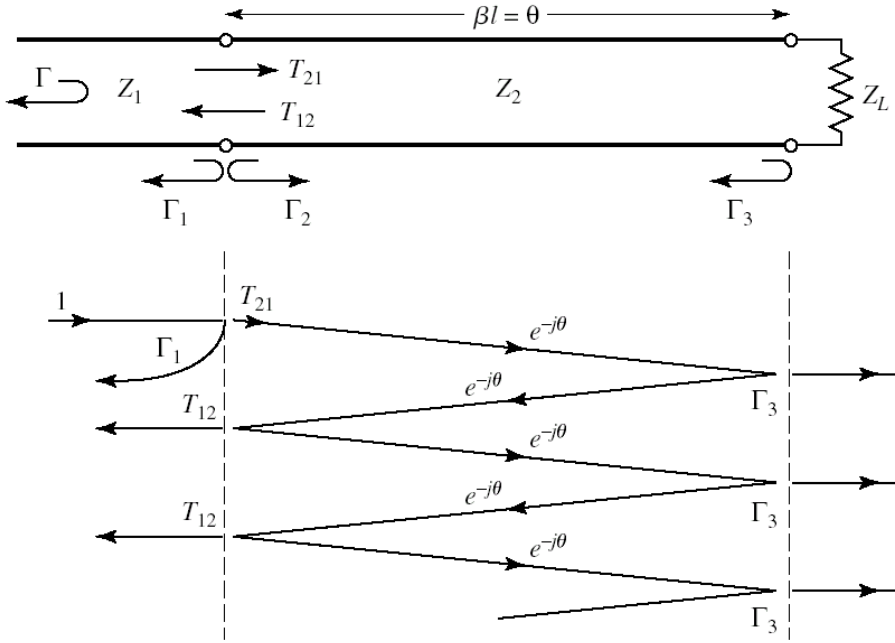
$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} \sum_{n=0}^{\infty} \Gamma_3^n \cdot \Gamma_2^n \cdot e^{-2jn\theta}$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad |x| < 1$$

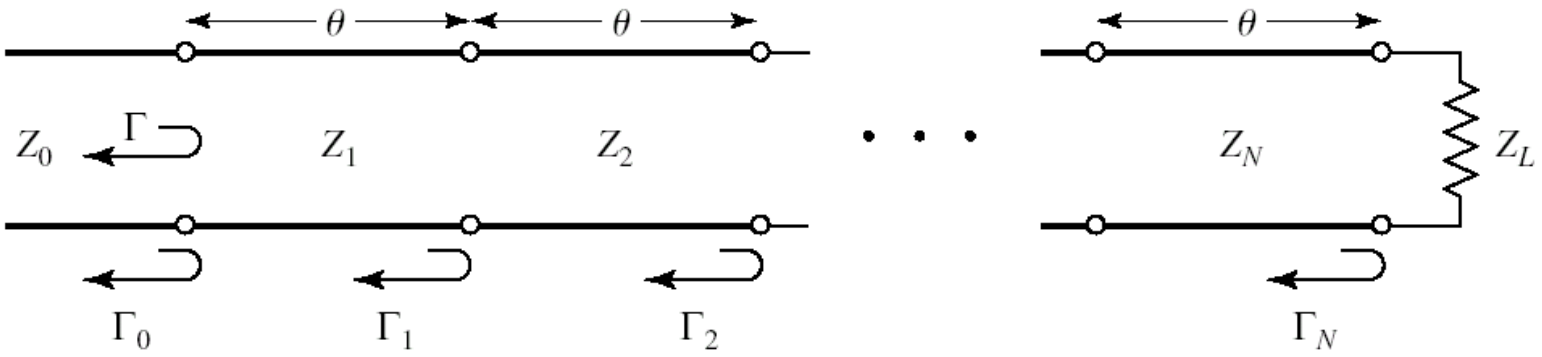
$$\Gamma = \frac{\Gamma_1 + \Gamma_3 \cdot e^{-2j\theta}}{1 + \Gamma_1 \cdot \Gamma_3 \cdot e^{-2j\theta}}$$

- Daca diferentele intre $Z_1 \div Z_2$ si $Z_2 \div Z_L$ sunt mici putem aproxima:

$$\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta}$$



Transformatoare cu mai multe sectiuni



- Presupunem ca toate impedantele **cresc sau descresc uniform**
- Toti coeficientii de reflexie vor fi reali si de acelasi semn
- Anterior $\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta} \Rightarrow$

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

$$n = \overline{1, N-1}$$

$$\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}$$

Transformatoare cu mai multe sectiuni

- Realizez transformatorul **simetric**

$$\Gamma_0 = \Gamma_N, \Gamma_1 = \Gamma_{N-1}, \Gamma_2 = \Gamma_{N-2} \cdots$$

- Aceasta **nu** implica faptul ca impedantele sunt egale

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma(\theta) = e^{-jN\theta} \cdot \left[\Gamma_0 \cdot \left(e^{jN\theta} + e^{-jN\theta}\right) + \Gamma_1 \cdot \left(e^{j(N-2)\theta} + e^{-j(N-2)\theta}\right) + \Gamma_2 \cdot \left(e^{j(N-4)\theta} + e^{-j(N-4)\theta}\right) + \cdots\right]$$

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot \left[\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \cdots \Gamma_n \cdot \cos(N-2n)\theta + \cdots\right]$$

ultimul termen:

$$\cdots \frac{1}{2} \cdot \Gamma_{N/2} \quad n \text{ par}$$

$$\cdots \Gamma_{(N-1)/2} \cdot \cos\theta \quad n \text{ impar}$$

Transformatoare cu mai multe sectiuni

- Coeficient de reflexie

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$e^{-2j\theta} \equiv x$$

$$f(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + \cdots + a_N \cdot x^N$$

- aleg coeficientii astfel incat sa obtin o variatie dorita (a polinomului)

Transformatoare cu mai multe sectiuni cu caracteristica binomiala

- Raspunsul acestui transformator este de tip maxim plat in jurul frecventei de adaptare
- Pentru N sectiuni se anuleaza primele N-1 derivate ale functiei |Γ(θ)|

$$f(x) = A \cdot (1+x)^N$$

$$\Gamma(\theta) = A \cdot \left(1 + e^{-2j\theta}\right)^N$$

$$|\Gamma(\theta)| = |A| \cdot \left|e^{-j\theta}\right|^N \cdot \left|e^{j\theta} + e^{-j\theta}\right|^N = 2^N \cdot |A| \cdot |\cos\theta|^N$$

$$l = \frac{\lambda}{4} \Rightarrow \theta = \beta \cdot l = \frac{\pi}{2} \qquad \left|\Gamma\left(\frac{\pi}{2}\right)\right| = 0; \quad \frac{d^n}{d\theta^n}|\Gamma(\theta)|_{\theta=\frac{\pi}{2}} = 0 \qquad n = \overline{1, N-1}$$

Transformatoare cu mai multe sectiuni cu caracteristica binomiala

- A, θ →o , liniile de lungime o, dispar

$$\Gamma(0) = 2^N \cdot A = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad A = 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0}$$

- dezvoltarea binomului

$$f(x) = (1+x)^N = C_N^0 + C_N^1 \cdot x + \cdots + C_N^n \cdot x^n + \cdots + C_N^N \cdot x^N$$

$$C_N^n = \frac{N!}{(N-n)!n!}$$

- Coeficientii de reflexie

$$\Gamma(\theta) = A \cdot \left(1 + e^{-2j\theta}\right)^N \qquad \Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma_n = A \cdot C_N^n$$

Transformatoare cu mai multe sectiuni cu caracteristica binomiala

- Proiectare, solutii aproximative

$$A = 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_n = A \cdot C_N^n$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} \cong \frac{1}{2} \ln \frac{Z_{n+1}}{Z_n}$$

$$\ln x \cong 2 \cdot \frac{x-1}{x+1} \qquad x \cong 1$$

$$\ln \frac{Z_{n+1}}{Z_n} \cong 2 \cdot \Gamma_n = 2 \cdot A \cdot C_N^n = 2 \cdot 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0} \cdot C_N^n \cong 2^{-N} \cdot C_N^n \cdot \ln \frac{Z_L}{Z_0}$$

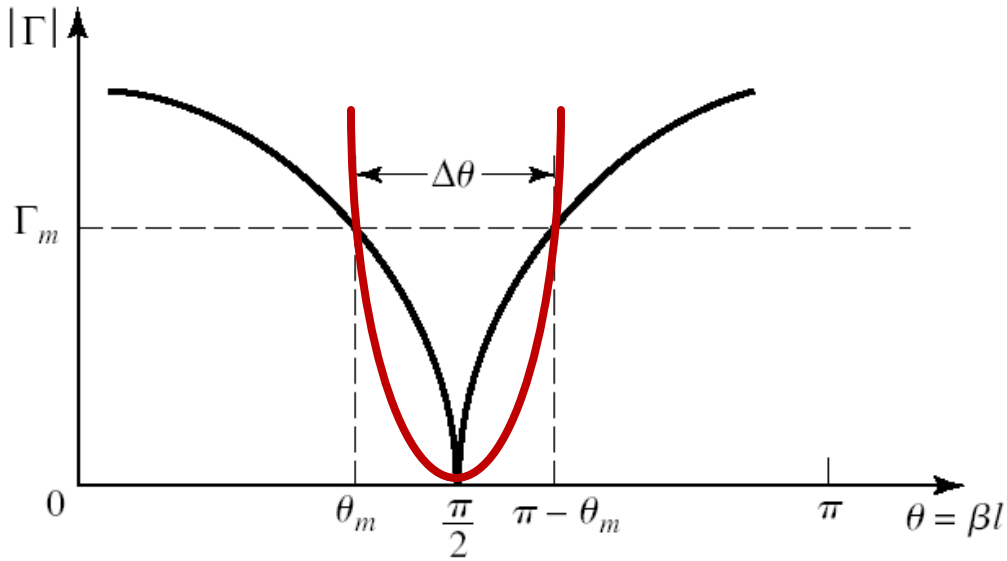
$$\ln Z_{n+1} \cong \ln Z_n + 2^{-N} \cdot C_N^n \cdot \ln \frac{Z_L}{Z_0}$$

Transformatoare cu mai multe sectiuni cu caracteristica binomiala

- Banda, Γ_m maxim tolerat

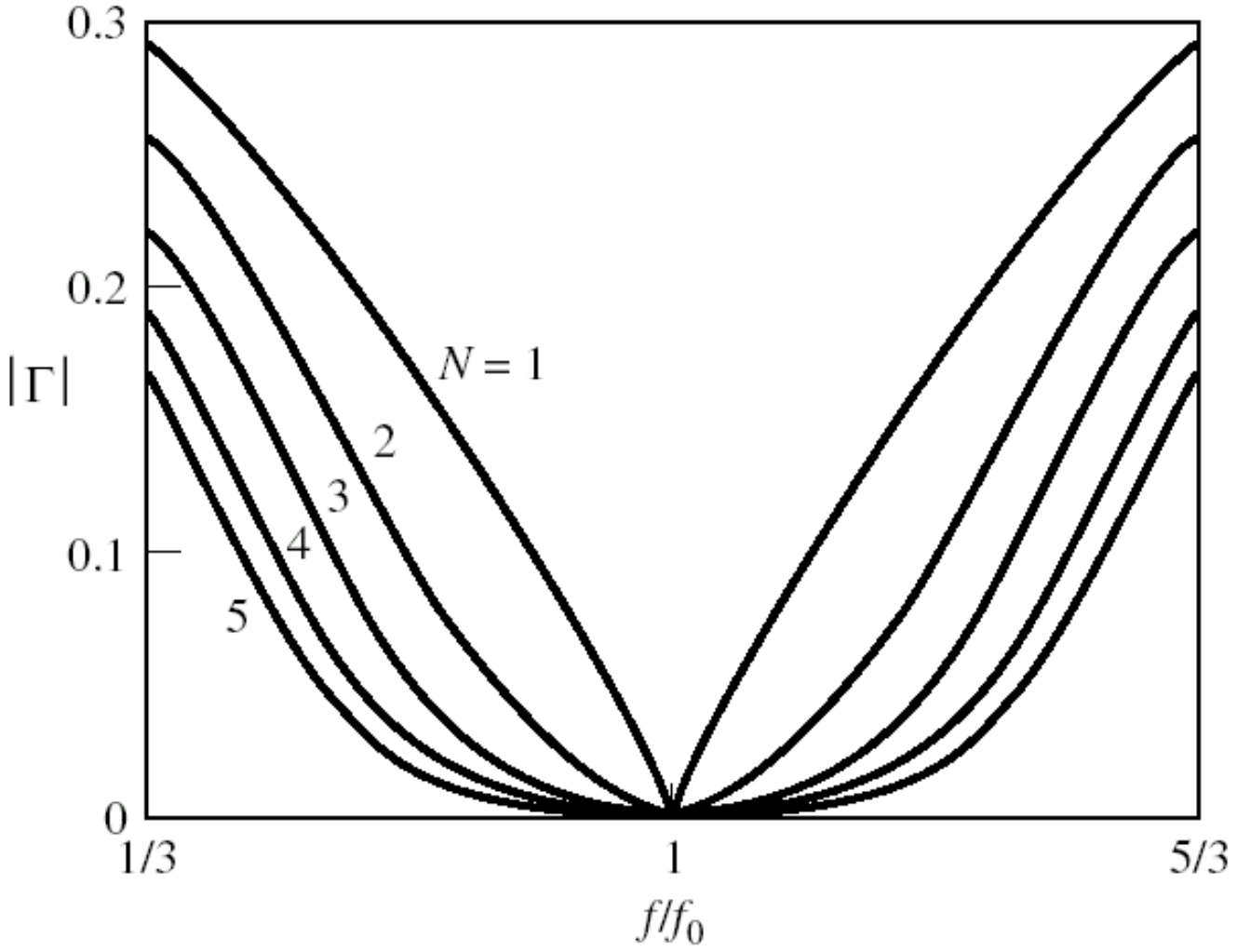
$$\Gamma_m = \left|\Gamma(\theta_m)\right| = 2^N \cdot |A| \cdot \left|\cos\theta_m\right|^N$$

$$\theta_m = \cos^{-1}\left[\frac{1}{2}\left(\frac{\Gamma_m}{|A|}\right)^{\frac{1}{N}}\right]$$



$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4}{\pi} \cdot \cos^{-1}\left[\frac{1}{2}\left(\frac{\Gamma_m}{|A|}\right)^{\frac{1}{N}}\right]$$

Banda



Transformatoare cu mai multe sectiuni cu caracteristica binomiala, rezultate exacte

	$N=2$		$N=3$			$N=4$			
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1067	1.3554	1.0520	1.2247	1.4259	1.0257	1.1351	1.3215	1.4624
2.0	1.1892	1.6818	1.0907	1.4142	1.8337	1.0444	1.2421	1.6102	1.9150
3.0	1.3161	2.2795	1.1479	1.7321	2.6135	1.0718	1.4105	2.1269	2.7990
4.0	1.4142	2.8285	1.1907	2.0000	3.3594	1.0919	1.5442	2.5903	3.6633
6.0	1.5651	3.8336	1.2544	2.4495	4.7832	1.1215	1.7553	3.4182	5.3500
8.0	1.6818	4.7568	1.3022	2.8284	6.1434	1.1436	1.9232	4.1597	6.9955
10.0	1.7783	5.6233	1.3409	3.1623	7.4577	1.1613	2.0651	4.8424	8.6110

	$N=5$					$N=6$					
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_6/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0128	1.0790	1.2247	1.3902	1.4810	1.0064	1.0454	1.1496	1.3048	1.4349	1.4905
2.0	1.0220	1.1391	1.4142	1.7558	1.9569	1.0110	1.0790	1.2693	1.5757	1.8536	1.9782
3.0	1.0354	1.2300	1.7321	2.4390	2.8974	1.0176	1.1288	1.4599	2.0549	2.6577	2.9481
4.0	1.0452	1.2995	2.0000	3.0781	3.8270	1.0225	1.1661	1.6129	2.4800	3.4302	3.9120
6.0	1.0596	1.4055	2.4495	4.2689	5.6625	1.0296	1.2219	1.8573	3.2305	4.9104	5.8275
8.0	1.0703	1.4870	2.8284	5.3800	7.4745	1.0349	1.2640	2.0539	3.8950	6.3291	7.7302
10.0	1.0789	1.5541	3.1623	6.4346	9.2687	1.0392	1.2982	2.2215	4.5015	7.7030	9.6228

Exemplu

- Transformator de adaptare cu 3 sectiuni pentru a adapta o sarcina de 30Ω la o linie de 100 Ω la frecventa f_0=3GHz, Γ_m=0.1
  - N = 3

$$Z_L = 30\Omega \qquad Z_0 = 100\,\Omega$$

$$A = 2^{-N}\frac{Z_L - Z_0}{Z_L + Z_0} \approx \frac{1}{2^{N+1}}\ln\frac{Z_L}{Z_0} = -0.07525$$

$$C_3^0 = \frac{3!}{3!\cdot 0!} = 1 \qquad C_3^1 = \frac{3!}{2!\cdot 1!} = 3 \qquad C_3^2 = \frac{3!}{1!\cdot 2!} = 3$$

Exemplu

$n = 0$

$$\ln Z_1 = \ln Z_0 + 2^{-N} C_3^0 \ln \frac{Z_L}{Z_0} = \ln 100 + 2^{-3} \cdot 1 \cdot \ln \frac{30}{100} = 4.455$$

$$Z_1 = 86.03\Omega$$

$n = 1$

$$\ln Z_2 = \ln Z_1 + 2^{-N} C_3^1 \ln \frac{Z_L}{Z_0} = \ln 86.03 + 2^{-3} \cdot 3 \cdot \ln \frac{30}{100} = 4.003$$

$$Z_2 = 54.77\Omega$$

$n = 2$

$$\ln Z_3 = \ln Z_2 + 2^{-N} C_3^2 \ln \frac{Z_L}{Z_0} = \ln 54.77 + 2^{-3} \cdot 3 \cdot \ln \frac{30}{100} = 3.552$$

$$Z_3 = 34.87\Omega$$

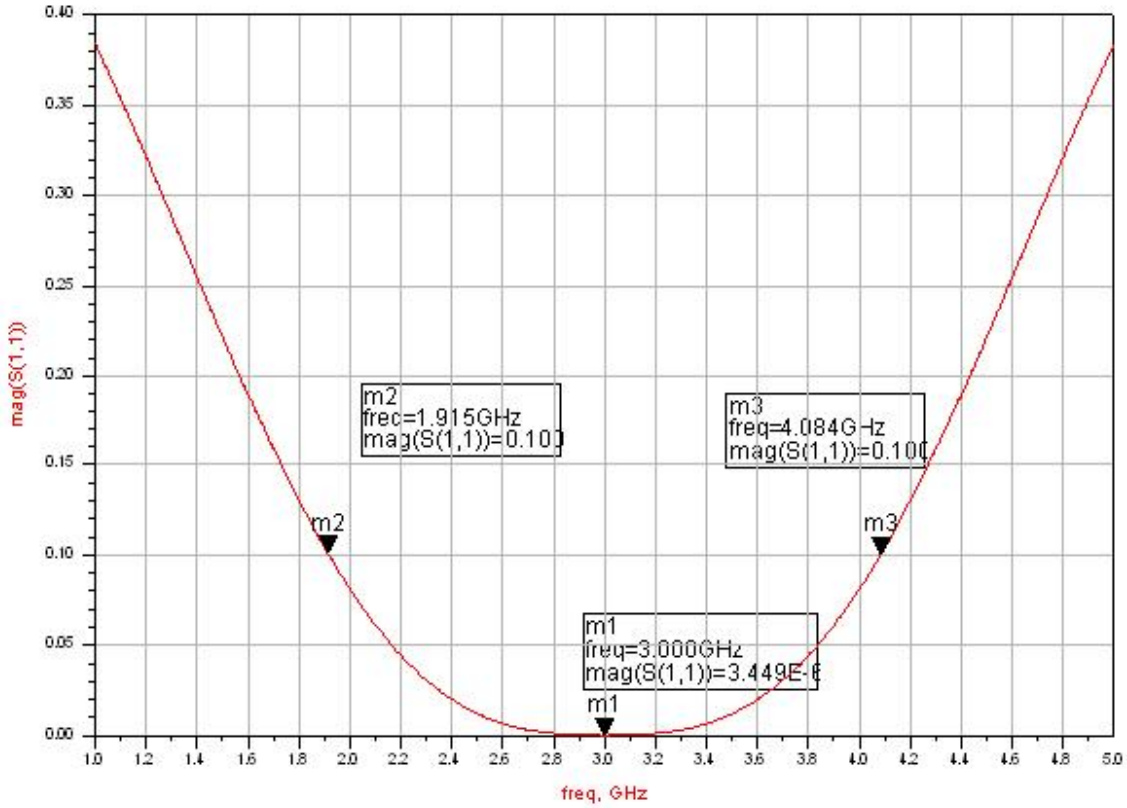
Exemplu

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\arccos\left[\frac{1}{2}\left(\frac{\Gamma_m}{|A|}\right)^{1/N}\right] = 2 - \frac{4}{\pi}\arccos\left[\frac{1}{2}\left(\frac{0.1}{0.07525}\right)^{1/3}\right] = 0.74$$

$$\Delta f = 2.22GHz$$

Simulare

- Similar Lab. 1



$$\Delta f = 2.169GHz$$

$$\left|\Gamma\left(3GHz\right)\right| = 3.5 \cdot 10^{-6}$$

Transformatoare cu mai multe sectiuni de tip Cebîşev

- Raspunsul acestui transformator este de tip echiriplu in jurul frecventei de adaptare
- mareste banda in detrimentul riplului (variatiilor) in banda de adaptare
- Se egaleaza functia $\Gamma(\theta)$ cu un polinom Cebîşev

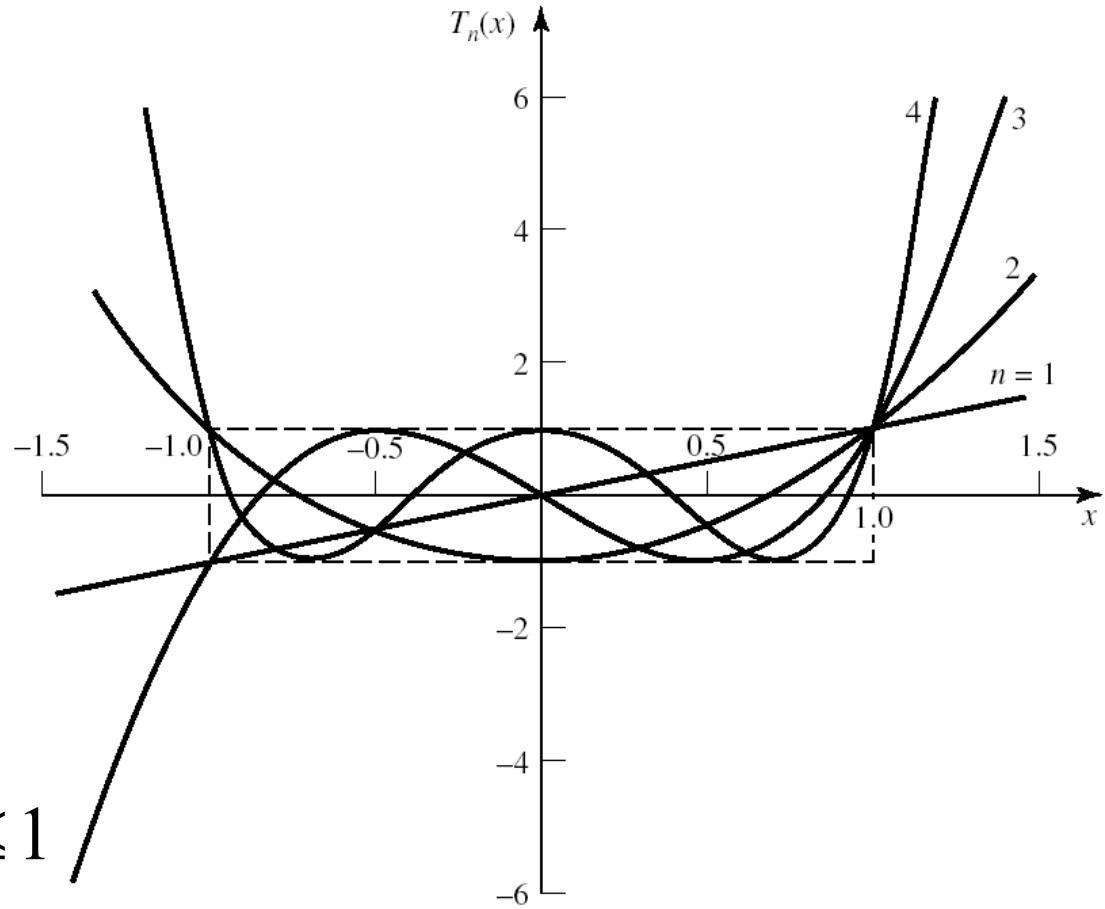
Polinoame Cebîşev

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$



- **Echiriplu**

$$-1 \le x \le 1 \quad \Rightarrow \quad |T_n(x)| \le 1$$

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$

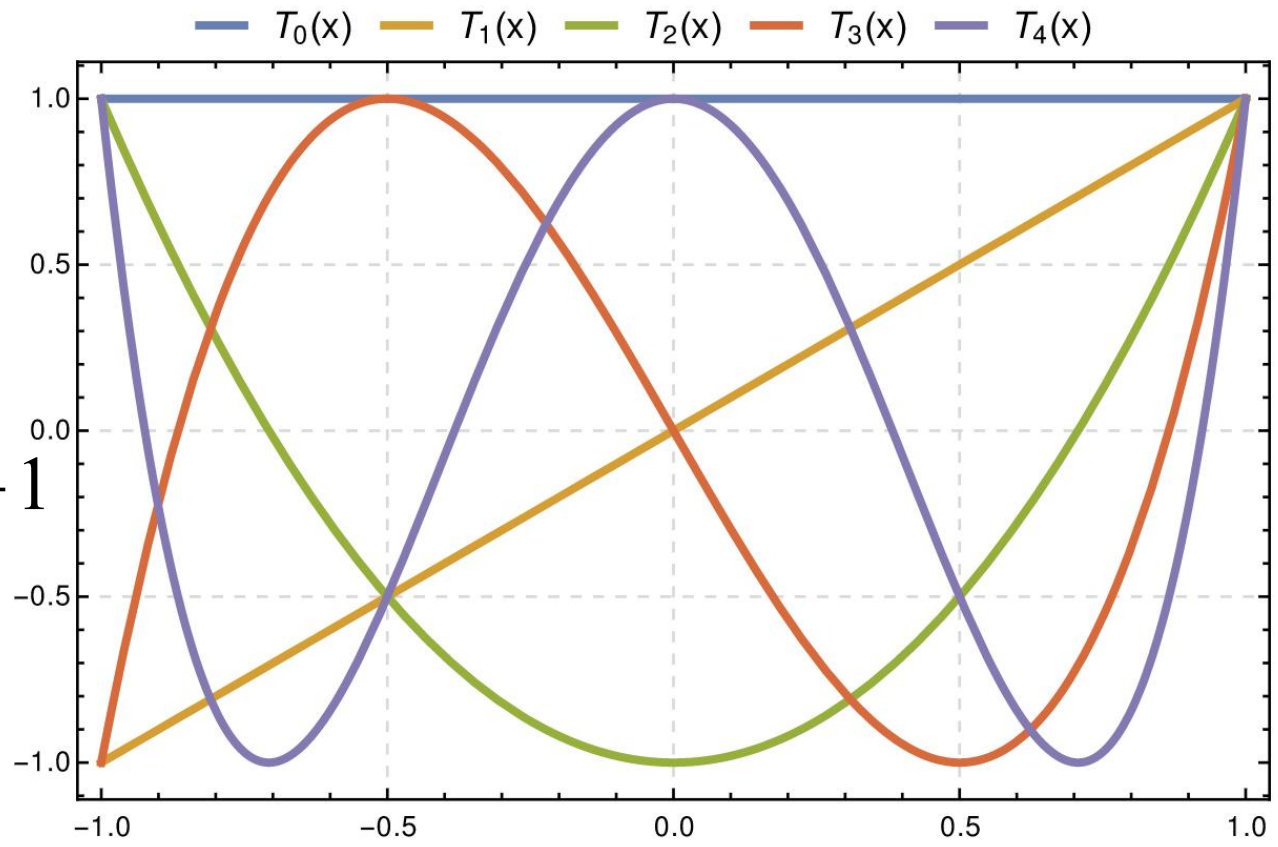
Polinoame Cebîşev

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$



- **Echiriplu**

$$-1 \le x \le 1 \quad \Rightarrow \quad |T_n(x)| \le 1$$

Polinoame Cebîşev

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$e^{-2j\theta} \equiv x$$

$$f(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + \cdots + a_N \cdot x^N$$

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot \left[\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \cdots \Gamma_n \cdot \cos(N-2n)\theta + \cdots\right]$$

ultimul termen:

$$\cdots \frac{1}{2} \cdot \Gamma_{N/2} \quad n \text{ par}$$

$$\cdots \Gamma_{(N-1)/2} \cdot \cos\theta \quad n \text{ impar}$$

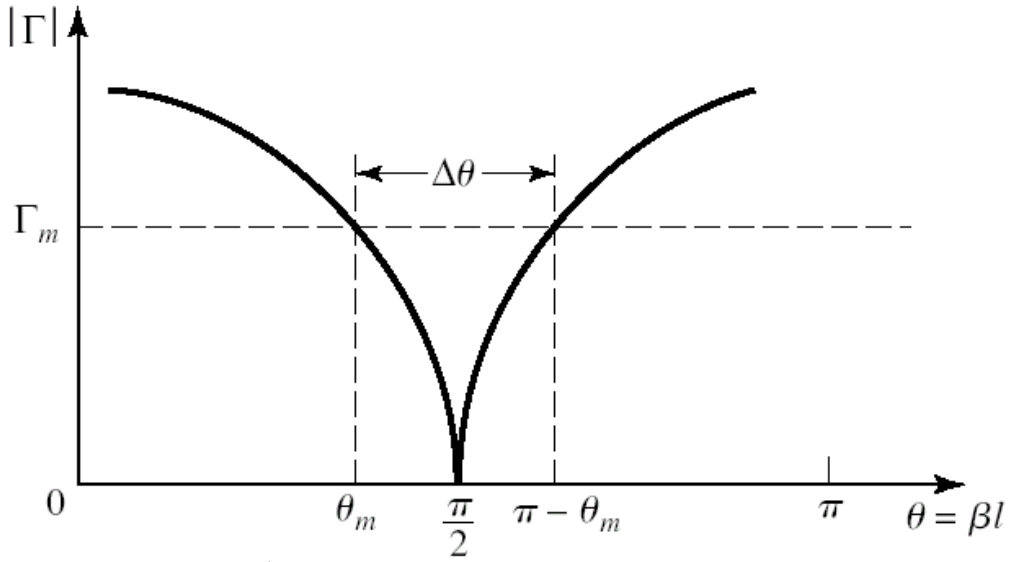
$$x = \cos\theta \qquad |x| < 1$$

- Se poate arata ca: $T_n(\cos\theta) = \cos(n\theta)$

$$T_n(x) = \cos(n \arccos(x)) \quad |x| < 1 \qquad T_n(x) = \cosh\left(n \cosh^{-1}(x)\right) \quad |x| > 1$$

Transformatoare cu mai multe sectiuni de tip Cebîşev

- Schimbare de variabila
  - banda de interes -> [-1,1]



$$\theta = \theta_m \rightarrow x = 1 \qquad \theta = \pi - \theta_m \rightarrow x = -1$$

$$x \equiv \frac{\cos\theta}{\cos\theta_m}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad x = \sec\theta_m \cos\theta$$

Transformatoare cu mai multe sectiuni de tip Cebîşev

- Cautam coeficientii pentru a obtine un polinom Cebîşev

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot \left[\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \cdots \Gamma_n \cdot \cos(N-2n)\theta + \cdots\right]$$

ultimul termen:

$$\cdots \frac{1}{2} \cdot \Gamma_{N/2} \quad n \text{ par}$$

$$\cdots \Gamma_{(N-1)/2} \cdot \cos\theta \quad n \text{ impar}$$

$$\Gamma(\theta) = A \cdot e^{-jN\theta} \cdot T_N\left(\sec\theta_m \cos\theta\right)$$

Polinoame Cebîşev

$(\cos\theta)^k \Leftrightarrow \cos k\theta$

$T_1(x) = x$ $\qquad$ $T_1(\sec\theta_m \cos\theta) = \sec\theta_m \cos\theta$

$T_2(x) = 2x^2 - 1$ $\qquad$ $T_2(\sec\theta_m \cos\theta) = 2\sec^2\theta_m \cos^2\theta - 1$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$$

$$T_2(\sec\theta_m \cos\theta) = \sec^2\theta_m (1 + \cos 2\theta) - 1$$

$T_3(x) = 4x^3 - 3x$ $\qquad$ $T_3(\sec\theta_m \cos\theta) = 4\sec^3\theta_m \cos^3\theta - 3\sec\theta_m \cos\theta$

$$\cos 3\theta = \cos(2\theta + \theta) = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$$

$$\cos 3\theta = (2\cos^2\theta - 1)\cos\theta - 2(1 - \cos^2\theta)\cos\theta$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$T_3(\sec\theta_m \cos\theta) = \sec^3\theta_m (\cos 3\theta + 3\cos\theta) - 3\sec\theta_m \cos\theta$$

Transformatoare cu mai multe sectiuni de tip Cebîşev

$$T_1\left(\sec\theta_m \cos\theta\right) = \sec\theta_m \cos\theta$$

$$T_2\left(\sec\theta_m \cos\theta\right) = \sec^2\theta_m \left(1+\cos 2\theta\right) - 1$$

$$T_3\left(\sec\theta_m \cos\theta\right) = \sec^3\theta_m \left(\cos 3\theta + 3\cos\theta\right) - 3\sec\theta_m \cos\theta$$

$$T_4\left(\sec\theta_m \cos\theta\right) = \sec^4\theta_m \left(\cos 4\theta + 4\cos 2\theta + 3\right) - 4\sec^2\theta_m \left(\cos 2\theta + 1\right) + 1$$

- Cautam coeficientii pentru a obtine un polinom Cebîşev

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot \left[\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \cdots \Gamma_n \cdot \cos(N-2n)\theta + \cdots\right]$$

$$\Gamma(\theta) = A \cdot e^{-jN\theta} \cdot T_N\left(\sec\theta_m \cos\theta\right)$$

ultimul termen:

$$\cdots \frac{1}{2} \cdot \Gamma_{N/2} \quad n \text{ par}$$

$$\cdots \Gamma_{(N-1)/2} \cdot \cos\theta \quad n \text{ impar}$$

Transformatoare cu mai multe sectiuni de tip Cebîşev

- A, θ →o , liniile de lungime o, dispar

$$\Gamma(0)=\frac{Z_L-Z_0}{Z_L+Z_0}=A\cdot T_N(\sec\theta_m) \qquad A=\frac{Z_L-Z_0}{Z_L+Z_0}\cdot\frac{1}{T_N(\sec\theta_m)}$$

$$\boxed{\Gamma_m=|A|}$$

$$T_N(\sec\theta_m)=\frac{1}{\Gamma_m}\left|\frac{Z_L-Z_0}{Z_L+Z_0}\right|\cong\frac{1}{2\Gamma_m}\left|\ln\frac{Z_L}{Z_0}\right|$$

$$T_n(x)=\cosh\left(n\cosh^{-1}(x)\right)$$

$$\sec\theta_m=\cosh\left[\frac{1}{N}\cosh^{-1}\left(\frac{1}{\Gamma_m}\left|\frac{Z_L-Z_0}{Z_L+Z_0}\right|\right)\right]\cong\cosh\left[\frac{1}{N}\cosh^{-1}\left(\left|\frac{\ln(Z_L/Z_0)}{2\Gamma_m}\right|\right)\right]$$

- se poate calcula θ_m pentru Γ_m maxim tolerat (riplu) deci banda

$$\frac{\Delta f}{f_0}=\frac{2(f_0-f_m)}{f_0}=2-\frac{4\theta_m}{\pi}$$

Transformatoare cu mai multe sectiuni de tip Cebîşev

- Proiectare, solutii aproximative

$$\Gamma_m = |A|$$

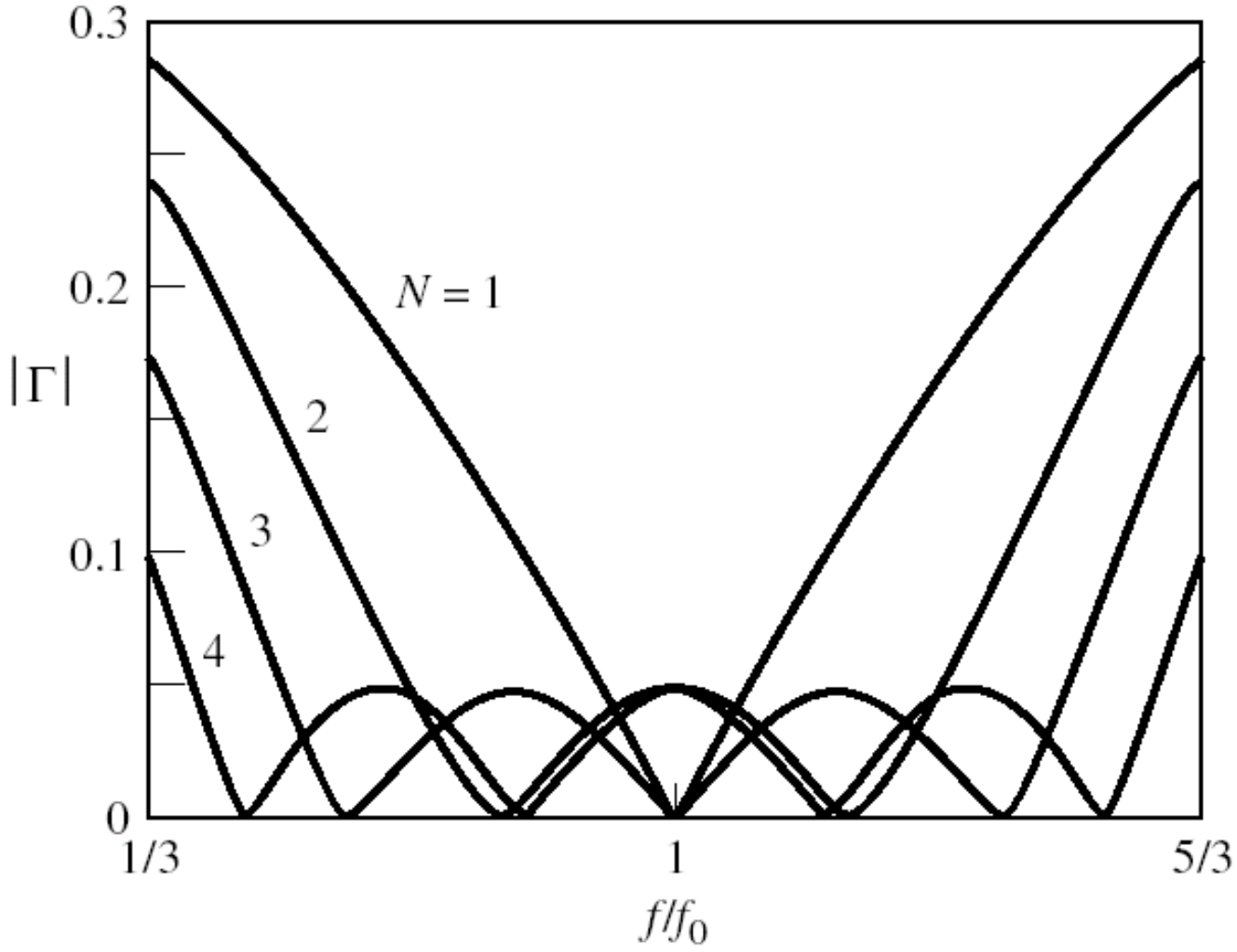
$$A = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot \frac{1}{T_N(\sec\theta_m)}$$

- Semnul lui A depinde de $Z_L <> Z_o$
- Se calculeaza sec θ_m
- Se reprezinta polinomul Cebîşev de ordin ales si se identifica coeficientii pentru cos kθ

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \cdots \Gamma_n \cdot \cos(N-2n)\theta + \cdots]$$

$$\ln\frac{Z_{n+1}}{Z_n} \cong 2 \cdot \Gamma_n \qquad \ln Z_{n+1} \cong \ln Z_n + 2 \cdot \Gamma_n$$

Transformatoare cu mai multe sectiuni de tip Cebîşev



Transformatoare cu mai multe sectiuni de tip Cebîşev, rezultate exacte

	$N = 2$				$N = 3$					
	$\Gamma_m = 0.05$		$\Gamma_m = 0.20$		$\Gamma_m = 0.05$			$\Gamma_m = 0.20$		
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1347	1.3219	1.2247	1.2247	1.1029	1.2247	1.3601	1.2247	1.2247	1.2247
2.0	1.2193	1.6402	1.3161	1.5197	1.1475	1.4142	1.7429	1.2855	1.4142	1.5558
3.0	1.3494	2.2232	1.4565	2.0598	1.2171	1.7321	2.4649	1.3743	1.7321	2.1829
4.0	1.4500	2.7585	1.5651	2.5558	1.2662	2.0000	3.1591	1.4333	2.0000	2.7908
6.0	1.6047	3.7389	1.7321	3.4641	1.3383	2.4495	4.4833	1.5193	2.4495	3.9492
8.0	1.7244	4.6393	1.8612	4.2983	1.3944	2.8284	5.7372	1.5766	2.8284	5.0742
10.0	1.8233	5.4845	1.9680	5.0813	1.4385	3.1623	6.9517	1.6415	3.1623	6.0920

$N = 4$

	$\Gamma_m = 0.05$				$\Gamma_m = 0.20$			
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0892	1.1742	1.2775	1.3772	1.2247	1.2247	1.2247	1.2247
2.0	1.1201	1.2979	1.5409	1.7855	1.2727	1.3634	1.4669	1.5715
3.0	1.1586	1.4876	2.0167	2.5893	1.4879	1.5819	1.8965	2.0163
4.0	1.1906	1.6414	2.4369	3.3597	1.3692	1.7490	2.2870	2.9214
6.0	1.2290	1.8773	3.1961	4.8820	1.4415	2.0231	2.9657	4.1623
8.0	1.2583	2.0657	3.8728	6.3578	1.4914	2.2428	3.5670	5.3641
10.0	1.2832	2.2268	4.4907	7.7930	1.5163	2.4210	4.1305	6.5950

Exemplu

- Transformator de adaptare cu 3 sectiuni pentru a adapta o sarcina de 30Ω la o linie de 100 Ω la frecventa f_0=3GHz, Γ_m=0.1
  - N = 3 $\quad Z_L = 30\Omega \quad Z_0 = 100\,\Omega$

$$\Gamma(\theta) = 2e^{-j3\theta}\left[\Gamma_0 \cos 3\theta + \Gamma_1 \cos\theta\right] = Ae^{-j3\theta}T_3\left(\sec\theta_m \cos\theta\right)$$

$$|A| = \Gamma_m = 0.1 \qquad A = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot \frac{1}{T_N\left(\sec\theta_m\right)} \qquad Z_L < Z_0 \rightarrow A < 0 \qquad A = -0.1$$

$$\sec\theta_m = \cosh\left[\frac{1}{N}\cdot\cosh^{-1}\left(\left|\frac{\ln Z_L/Z_0}{2\Gamma_m}\right|\right)\right] = \cosh\left[\frac{1}{3}\cdot\cosh^{-1}\left(\left|\frac{\ln(30/100)}{2\cdot 0.1}\right|\right)\right] = 1.362$$

$$\theta_m = \arccos\left(\frac{1}{\sec\theta_m}\right) = 0.746\,rad = 42.76°$$

Exemplu

$$2\left[\Gamma_0 \cos 3\theta + \Gamma_1 \cos\theta\right] = A\sec^3\theta_m\left(\cos 3\theta + 3\cos\theta\right) - 3A\sec\theta_m \cos\theta$$

$\cos 3\theta$ $\qquad 2\Gamma_0 = A\sec^3\theta_m$ $\qquad \Gamma_0 = -0.1263$

$\cos\theta$ $\qquad 2\Gamma_1 = 3A\left(\sec^3\theta_m - \sec\theta_m\right)$ $\qquad \Gamma_1 = -0.1747$

simetrie: $\Gamma_3 = \Gamma_0;\quad \Gamma_2 = \Gamma_1$

Exemplu

$n = 0$

$$\ln Z_1 = \ln Z_0 + 2 \cdot \Gamma_0 = \ln 100 - 2 \cdot 0.1263 = 4.353 \qquad \Gamma_0 = -0.1263$$

$$Z_1 = 77.68\Omega \qquad \Gamma_1 = -0.1747$$

$n = 1$

$$\ln Z_2 = \ln Z_1 + 2 \cdot \Gamma_1 = \ln 77.68 - 2 \cdot 0.1747 = 4.003$$

$$Z_2 = 54.77\Omega$$

$n = 2$

$$\ln Z_3 = \ln Z_2 + 2 \cdot \Gamma_2 = \ln 54.77 - 2 \cdot 0.1747 = 3.654$$

$$Z_3 = 38.62\Omega$$

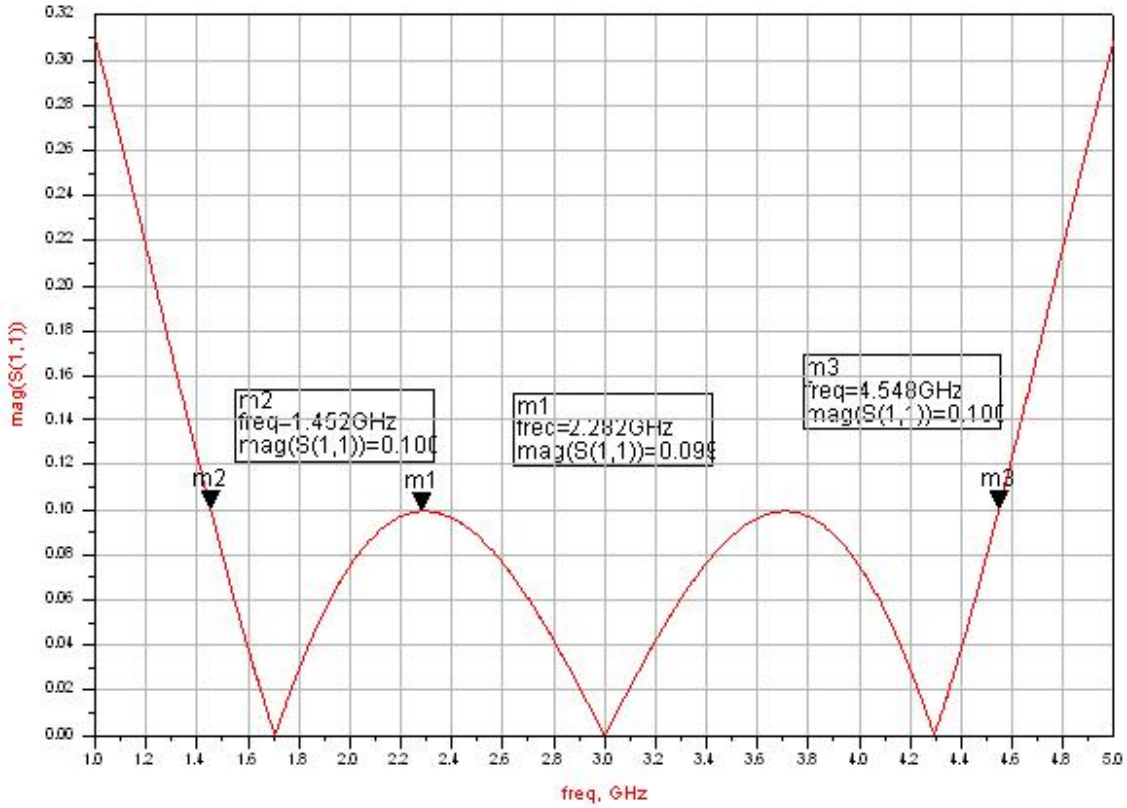
Exemplu

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4 \cdot 42.76^\circ}{180^\circ} = 1.045$$

$$\Delta f = 3.15 GHz$$

Simulare

- Similar Lab. 1



$$\Delta f = 3.096GHz$$

$$\left|\Gamma\left(3GHz\right)\right| = 4.17 \cdot 10^{-5}$$

$$\left|\Gamma\left(2.282GHz\right)\right| = 0.09925$$

Solutii exacte

- G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks,and Coupling Structures, Artech House Books, Dedham, Mass. 1980*

Puncte suplimentare

Tema

Tema

- Scrieti pe o foaie de hartie (si scanati/foto) relatiile pentru:
  - determinarea conditiei de maxim de putere utilizand derivate partiale (impedante complexe)

$$\frac{\partial P_L}{\partial R_L} = 0 \qquad \frac{\partial P_L}{\partial X_L} = 0$$

  - determinarea expresiei pentru polinomul Cebîşev de ordinul 4

$$T_4\left(\sec\theta_m \cos\theta\right) = \sec^4\theta_m\left(\cos 4\theta + 4\cos 2\theta + 3\right) - 4\sec^2\theta_m\left(\cos 2\theta + 1\right) + 1$$

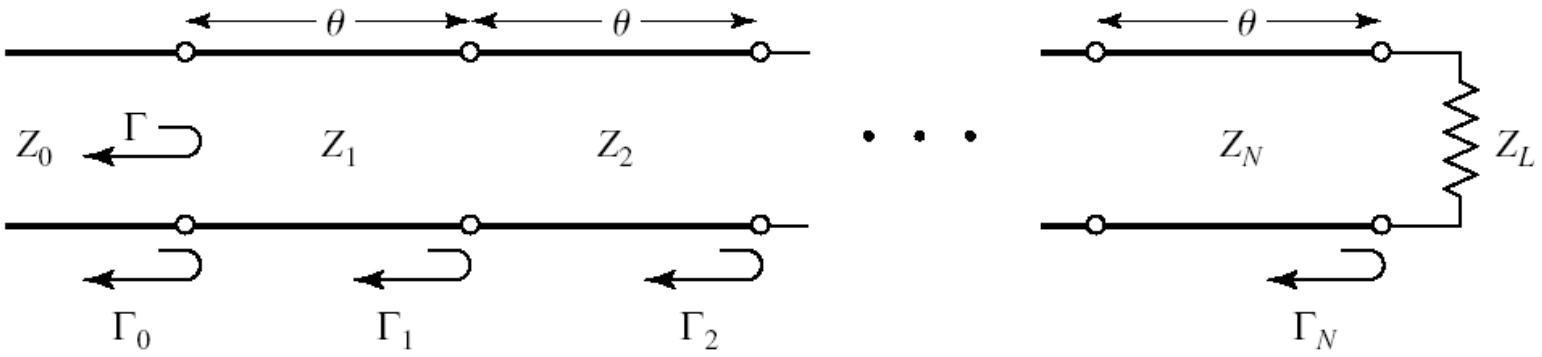
Laborator 2

2025/2026

Dispozitive şi circuite de microunde pentru radiocomunicații

Scurta teorie

Transformatoare cu mai multe sectiuni



- Presupunem ca toate impedantele **cresc sau descresc uniform**
- Toti coeficientii de reflexie vor fi reali si de acelasi semn
- Anterior $\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta} \Rightarrow$

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

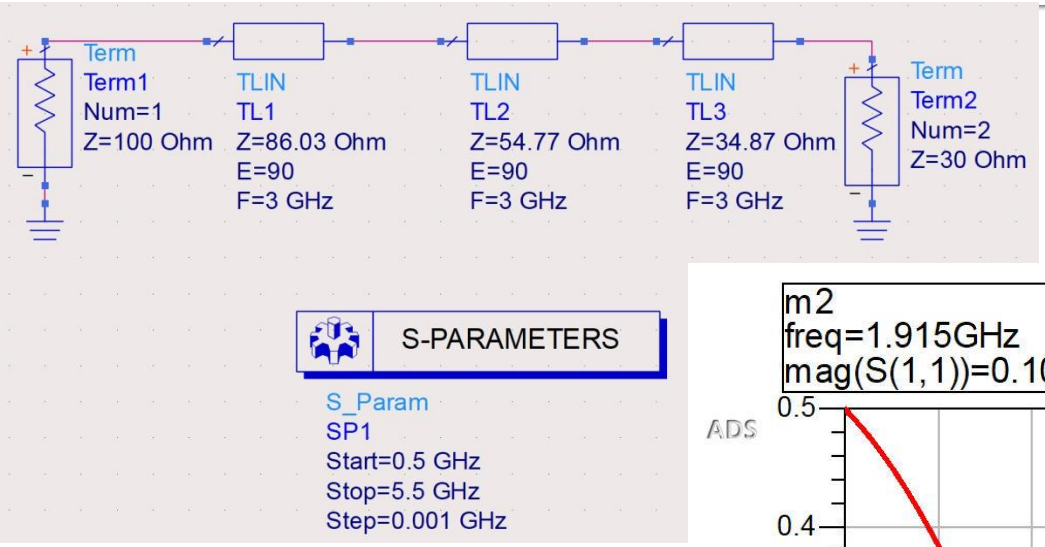
$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

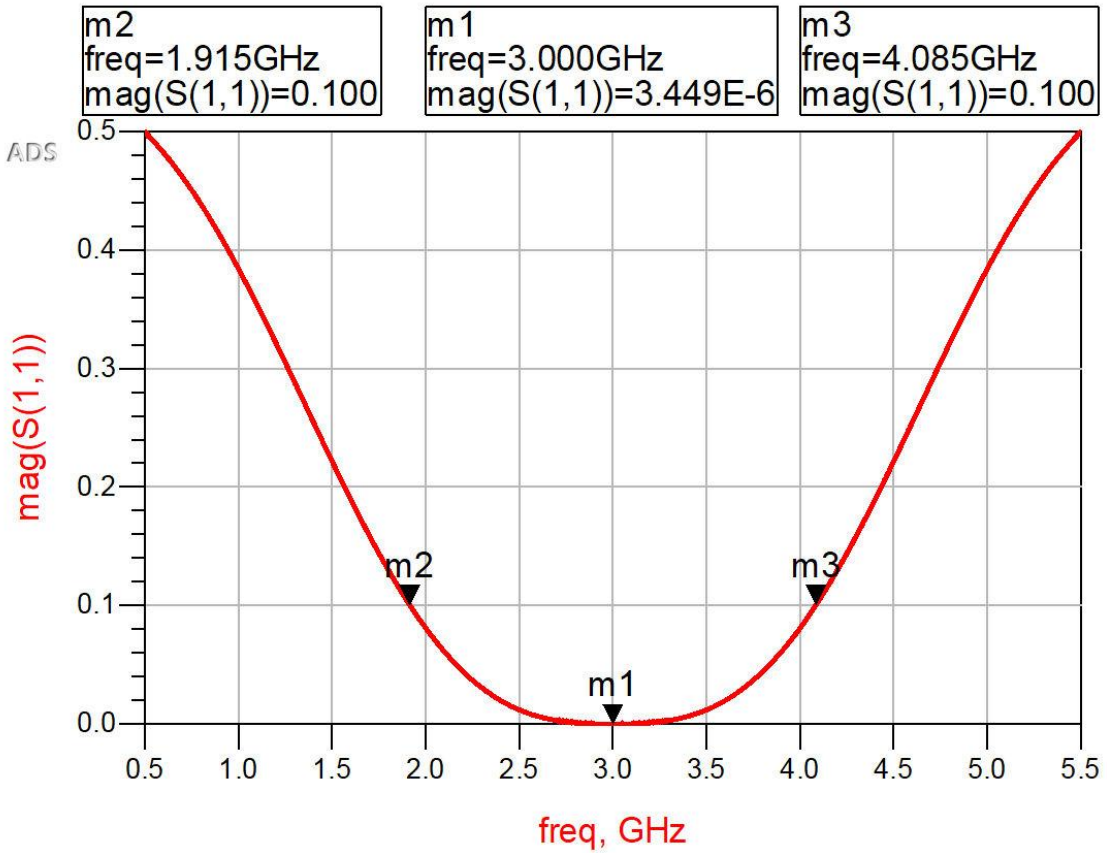
$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

$$n = \overline{1, N-1}$$

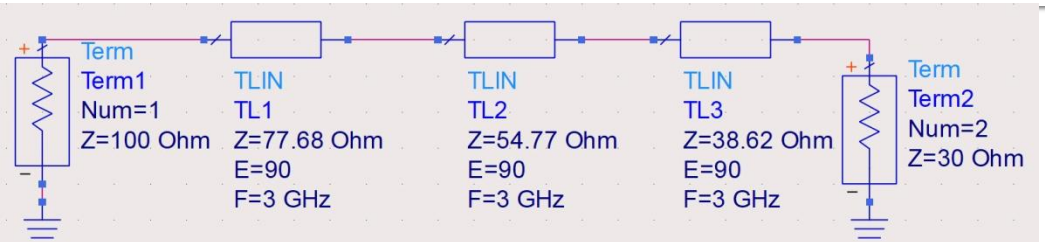
$$\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}$$

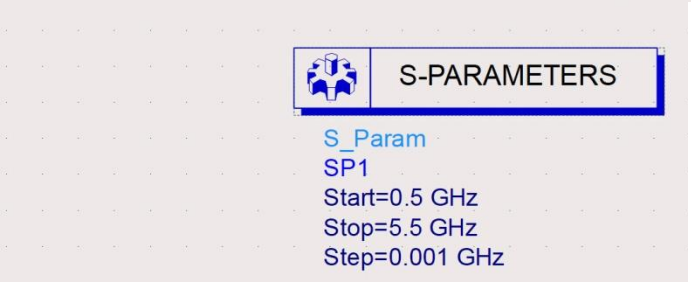
Binomial

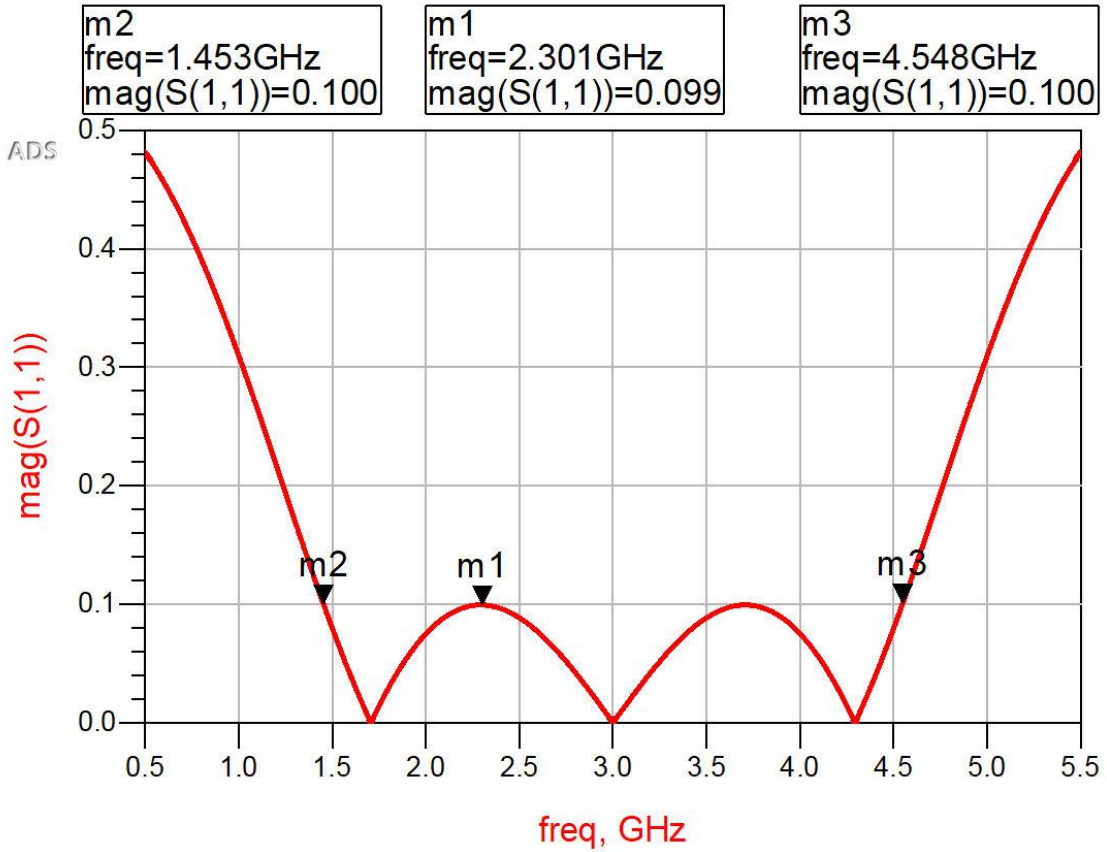

Term
Term1
Num=1
Z=100 Ohm
TLIN
TL1
Z=86.03 Ohm
E=90
F=3 GHz
TLIN
TL2
Z=54.77 Ohm
E=90
F=3 GHz
TLIN
TL3
Z=34.87 Ohm
E=90
F=3 GHz
Term
Term2
Num=2
Z=30 Ohm
S-PARAMETERS
S_Param
SP1
Start=0.5 GHz
Stop=5.5 GHz
Step=0.001 GHz


m2
freq=1.915GHz
mag(S(1,1))=0.100
m1
freq=3.000GHz
mag(S(1,1))=3.449E-6
m3
freq=4.085GHz
mag(S(1,1))=0.100
ADS
mag(S(1,1))
freq, GHz
0.0
0.1
0.2
0.3
0.4
0.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
m1
m2
m3

Cebâşev

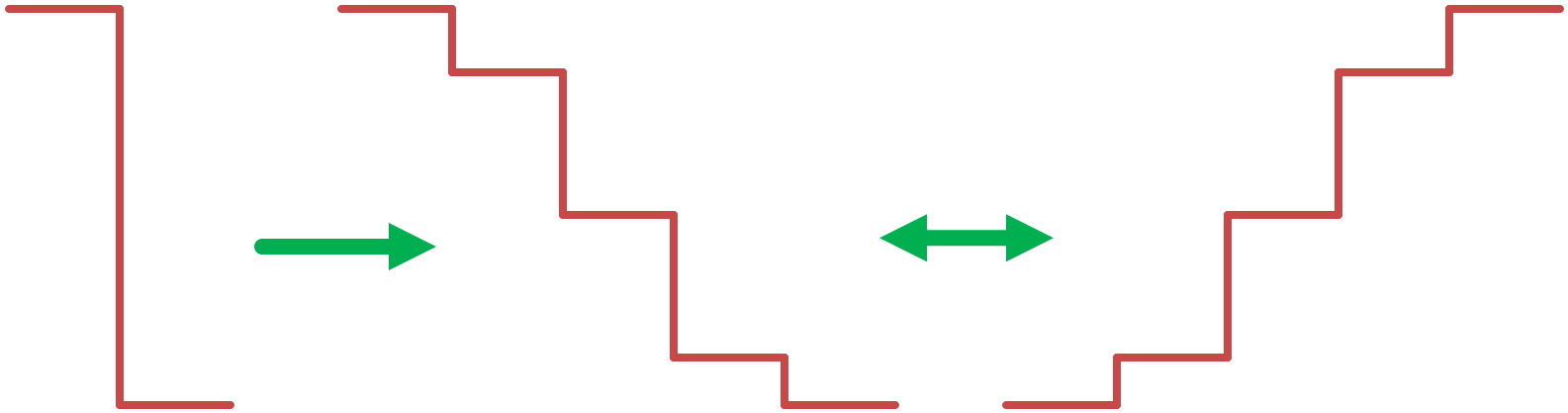

Term
Term1
Num=1
Z=100 Ohm
TLIN
TL1
Z=77.68 Ohm
E=90
F=3 GHz
TLIN
TL2
Z=54.77 Ohm
E=90
F=3 GHz
TLIN
TL3
Z=38.62 Ohm
E=90
F=3 GHz
Term
Term2
Num=2
Z=30 Ohm


S-PARAMETERS
S_Param
SP1
Start=0.5 GHz
Stop=5.5 GHz
Step=0.001 GHz


m2
freq=1.453GHz
mag(S(1,1))=0.100
m1
freq=2.301GHz
mag(S(1,1))=0.099
m3
freq=4.548GHz
mag(S(1,1))=0.100
ADS
mag(S(1,1))
freq, GHz
0.0
0.1
0.2
0.3
0.4
0.5
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
m2
m1
m3

Adaptare de impedanta

- Principial se inlocuieste o **variatie in salt** a impedantei cu o **crestere/descrestere graduala** a impedantei
- Pasii de crestere/descrestere graduala a impedantei trebuiesc atent calculati



Contact

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